Linear Algebra

Introduction

Department of Computer Engineering

Sharif University of Technology

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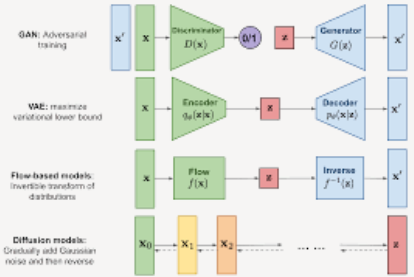
Course Description

How is our course?

Machine Learning Mathematics



Machine Learning Mathematics

01
Linear Algebra

Provides the foundation for manipulating data in high-dimensional spaces, essential for vector operations in machine learning.

02
Statistics & Probability

Offers tools to model uncertainty and make inferences about data, forming the backbone of many machine learning algorithms.

03
Optimization

Focuses on finding the best parameters for a model by minimizing or maximizing an objective function.

04
Information Theory

Analyzes data compression and transmission, helping quantify uncertainty and information gain in machine learning models.

05
Signal Processing

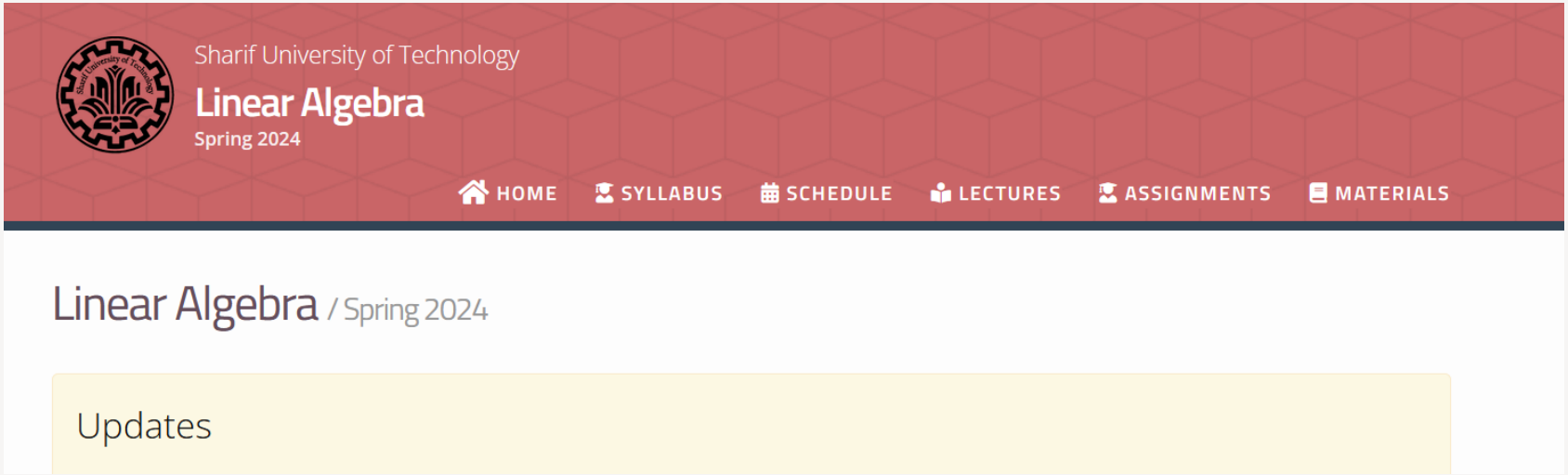
Involves analyzing and transforming data signals, important for handling time-series data and feature extraction in machine learning.

06
Graph Theory

Studies networks of nodes and edges, crucial for understanding relationships in data and complex models like Graph Neural Networks.

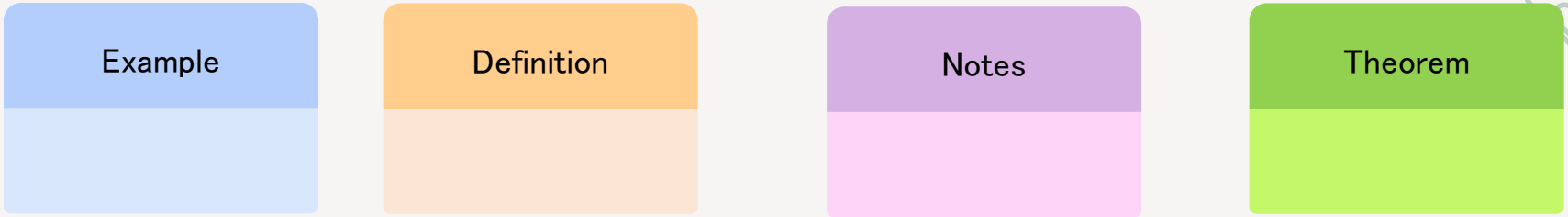
Course Material

https://sut-ce-courses.github.io/linearalgebra (jabrekh.ir)



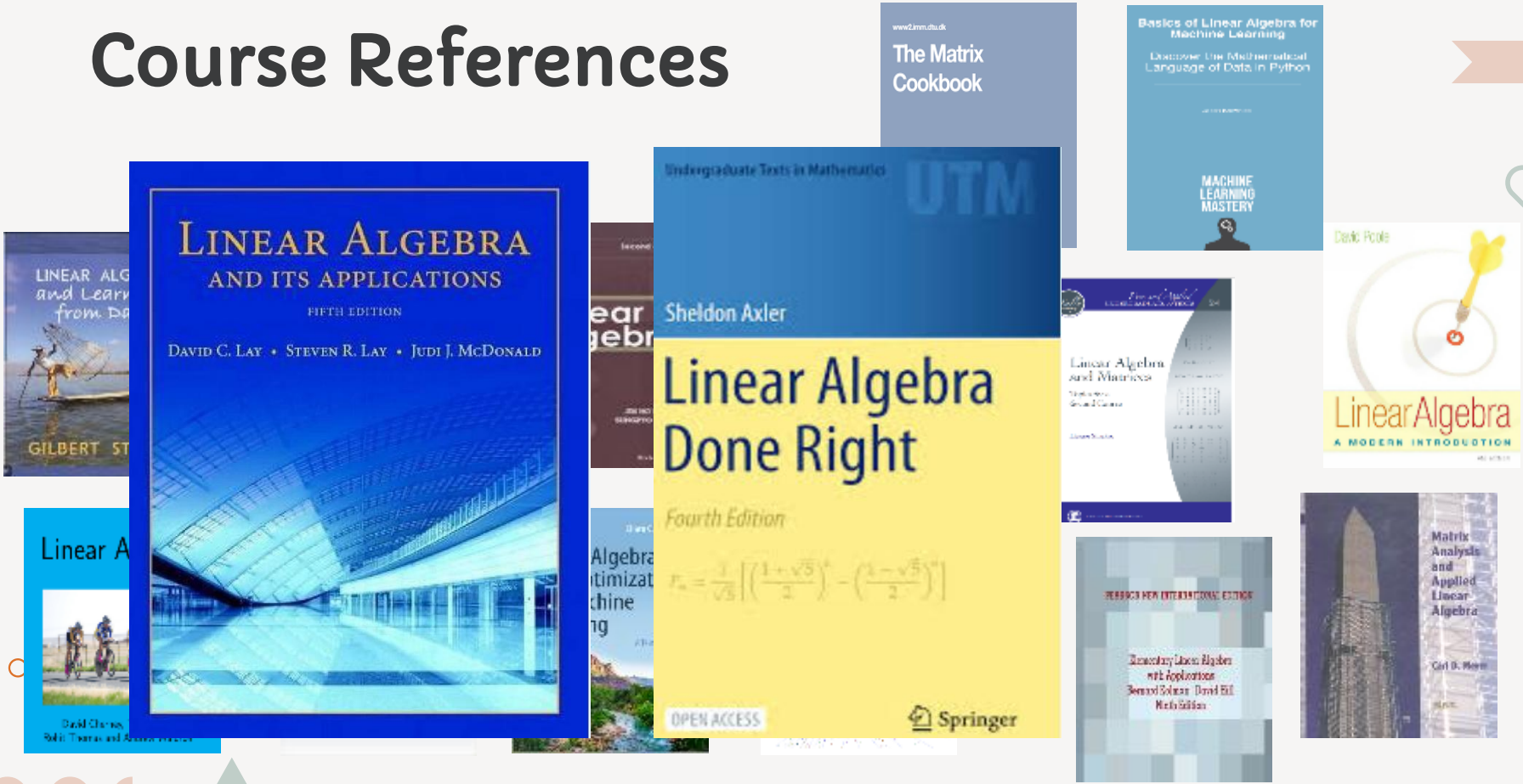
Course Method

- Slides



- Writing on board
- Geometric Interpretation and Intuition
- Notebooks

Course References



Reach Us!

- Office Room: CE802-CE803
- Email:
  - Hamid R. Rabiee: (rabiee@sharif.edu)
  - Maryam Ramezani: (maryam.ramezani@sharif.edu)
- Course notes, homework and solutions, handouts, and other useful resources are available on the course page:
  - https://quera.org/course/20776/
  - Room: (Sunday & Tuesday: 13:30-15:00)
  - Ebnesina Alef 11
  - https://vc.sharif.edu/ch/maryam.ramezani & https://vc.sharif.edu/ch/rabiee
- Lead TAs:


Shayan Shabani

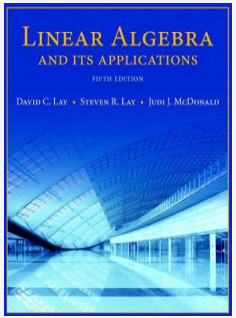

Amirmahdi Meighani

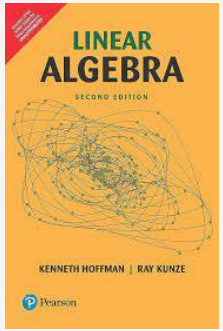
- Feedback
  - https://docs.google.com/forms/d/14A9BQNCtp4FfOCgPwvwAclcH9h3wSgyp0Tc8P7Uia50

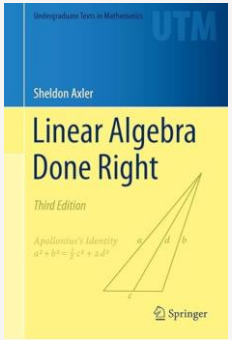
Resources

- Textbooks:
  - Sheldon Axler, Linear Algebra Done Right, fourth edition, 2023
  - Kenneth Hoffman and Ray A. Kunze.Linear Algebra. PHI Learning, 2004.
  - Gilbert Strang. Introduction to Linear Algebra. Wellesley–Cambridge Press, 2016.
  - David C. Lay, Steven R. Lay, and Judi J. McDonald.Linear Algebra and Its Applications. Pearson, 2016.

+Other textbooks and course materials.







Structure of the Course

Lectures

Goal: To introduce concepts in linear algebra, and motivate their use and importance.

Note: We try to cover useful materials in class, but we recommend you reading more!

Exams

Quiz: 5 with dropping lowest

Midterm: 1

Final: 1

Assignments

Purpose: To give you a chance to exercise your mind, and to solidify the concepts introduced to you in class.

Structure: Six theorical problems, and Three linear algebra practical problems.

Importance: Not important unless you want to learn the material and get a good grade.

Lecture Notes

- Lecture slide will be uploaded.
- Many times we will write on board, in real-time, during lecture to prove a theory or answer a question or add some additional explanations. It will be your responsibility to take notes.
- Slides links will be provided on site.

Syllabus

1403/11/21	S01	1	Elementary Row Operations, and Linear Equations
1403/11/23	S02		Row Reduction and Echelon Forms
1403/11/28	S03		Vector Space
1403/11/30	S04		Subspace

1403/12/05	S05	2	Independence (Linear and Affine)
1403/12/07	S06		Independence (Linear and Affine)
1403/12/12	S07		Bases, Dimension
1403/12/14	S08		Dimension and Rank
1403/12/19	S09		Dimension and Rank

Syllabus

1403/12/21	S10	3	Inner Product Space
1403/12/26	S11		Inner Product Space
1404/01/17	S12		Inequalities and Orthogonality
1404/01/19	S13		Orthogonality (Gram–Schmidt, etc.)

1404/01/24	S14	4	Linear Transformation
1404/01/26	S15		Linear Transformation
1404/01/31	S16		Change Basis
1404/02/02	S17		Inverse
1404/02/07	S18		Determinant
1404/02/09	S19		Determinant

Syllabus

1404/02/14	S20		Eigenvectors and Eigenvalues
1404/02/16	S21		Singular Values and Singular Vectors
1404/02/18	Midterm Exam		
1404/02/21	S22	5	Symmetric Matrices and Quadratic Forms
1404/02/23	S23		Diagonalization
1404/02/28	S24		Matrix Factorization
1404/02/30	S25		SVD

Syllabus

1404/03/04	S26	6	Norm Space
1404/03/06	S27		Derivatives
1404/03/11	S28		Derivatives
1404/03/13	S29		Least squares
1404/04/01	Final Exam		

Assignments

- Homework released at the beginning of each part and your responses should be uploaded on Quera.

- Programming Assignments: 3 points (0.5, 0.75, 0.75, 1 points respectively)

Assignments

	Release	Dealine
T1	1403/11/23	1403/12/05
P0	1403/11/28	1403/12/07
T2	1403/12/05	1403/12/21
T3	1403/12/21	1404/01/24
P1	1403/12/26	1404/01/26
T4	1404/01/24	1404/02/14
T5	1404/02/14	1404/03/04
P2	1404/02/16	1404/03/06
T6	1404/03/04	1404/03/18
P3 (porject)	1404/03/06	1404/04/01

Exams

Quizzes will be held as the following table on 13:00 for half an hour and will be graded in Grade Scope.

Exam	Time
Quiz 1	1403/12/14 13:00-13:30
Quiz 2	1404/01/17 13:00-13:30
Quiz 3	1404/01/31 13:00-13:30
Midterm	1404/02/18 15:00-18:00
Quiz 4	1404/03/06 13:00-13:30
Quiz 5	1404/03/13 13:00-13:30
Final	1404/04/01 09:00-12:00

TA Classes

TA Classes will be held on Wednesday from 18:00 till 20:00 in room https://vc.sharif.edu/maryam.ramezani.

Date	Title
1403/12/12	Part1
1404/01/12	Part2
1404/01/27	Part3
1404/02/17	Midterm
1404/02/31	Part4
1404/03/07	Part5
1404/03/28	Final

Grading Policy

- Quizes: 6 points (5 quizes each 1.5 points; the lowest score will be dropped)
- Programming Assignments: 3 points (0.5, 0.75, 0.75, 1 points respectively)
- Midterm Exam: 6 points (Lectures part 1,2,3,4)
- Final Exam: 6 points (Lectures part 5,6)
- Total Points: 21 points

Total: 21 Points

Introduction

Lets think about a question!

01

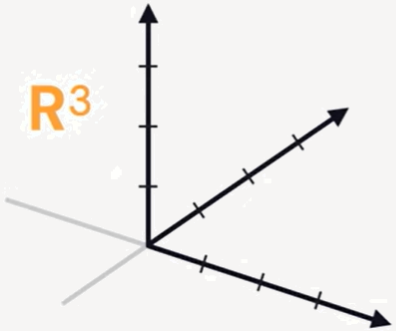
Vector

Basic Concept

Data Representations (Linear Algebra)

- How can we represent data (images, text, user preferences, etc.) in a way that computers can understand?
- Organize information into a vector!
  - A vector is a 1-dimensional array of numbers.
  - It has both a magnitude (length) and a direction
- The totality of a vectors with n entries is an n-dimensional vector space

$$V = \nearrow = \begin{bmatrix} -3 \\ 0.7 \\ 2 \end{bmatrix}$$



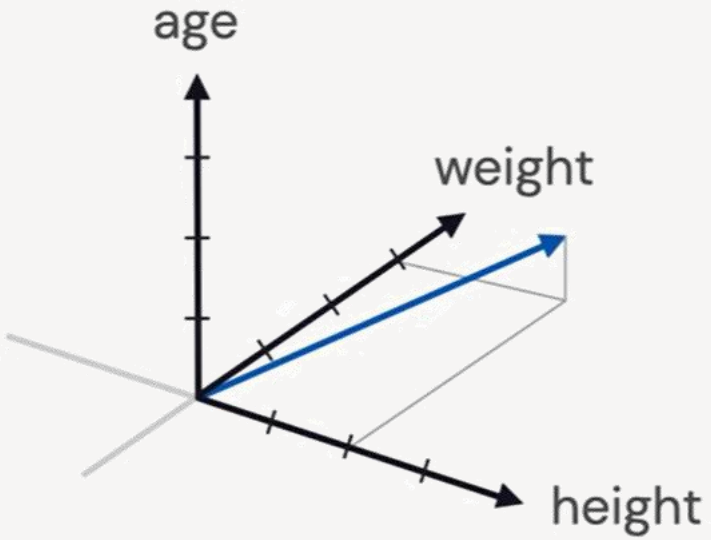
"3-dimensional space" consists of all vectors with 3 entries:

$$\begin{bmatrix} * \\ * \\ * \end{bmatrix}$$

Data Representations (Machine Learning)

- A **feature vector** is a vector whose entries represent the “features” of an object.
- The vector space containing them is called **feature space**.

$$P = \begin{bmatrix} 64 \\ 131 \\ 24 \end{bmatrix} \begin{matrix} \text{height} \\ \text{weight} \\ \text{age} \end{matrix}$$

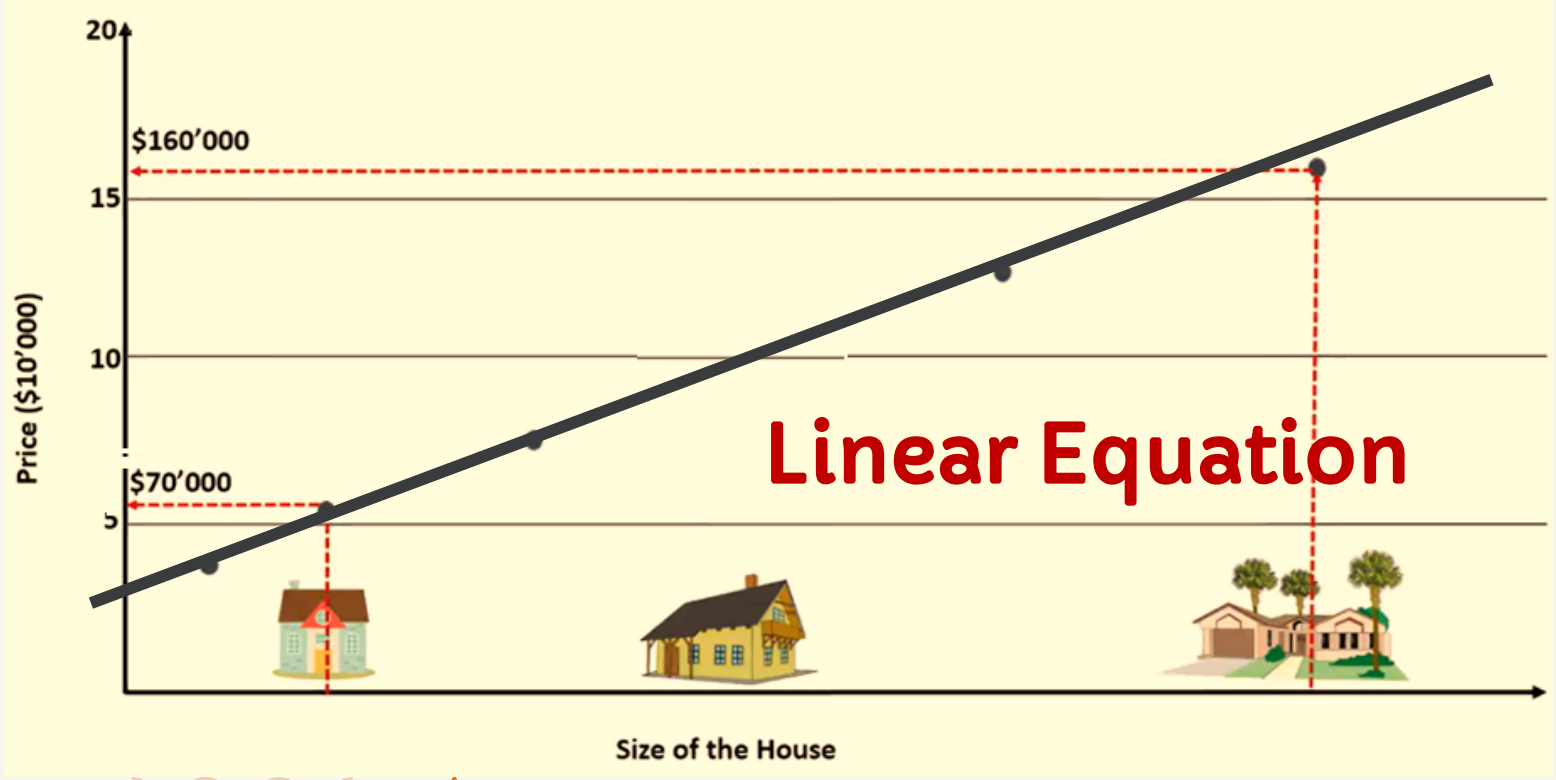


02
Equation with Matrix and Vector Format

House Pricing Example

Price Problem

Price Problem



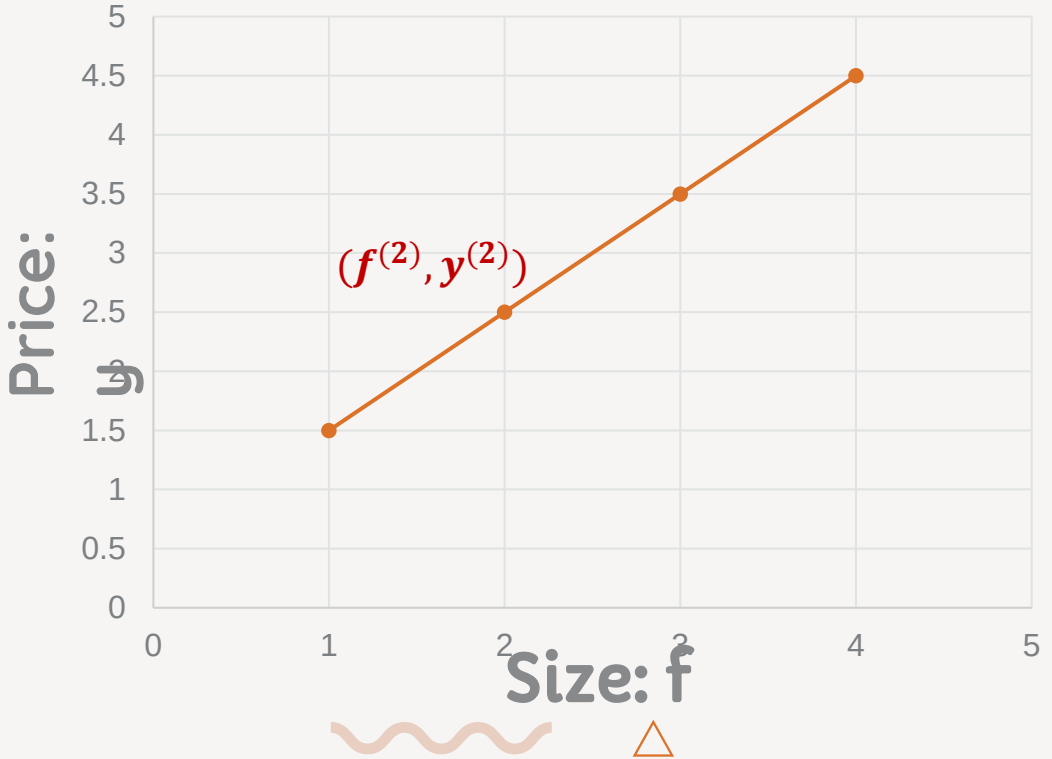
Linear Equation without offset

$$y = af$$

How to convert it to matrix-vecto multiplication?

$$Ax = b$$

Linear Equation with offset



$$y = af + c$$

How to convert it to matrix-vecto multiplication?

$$Ax = b$$

House Features

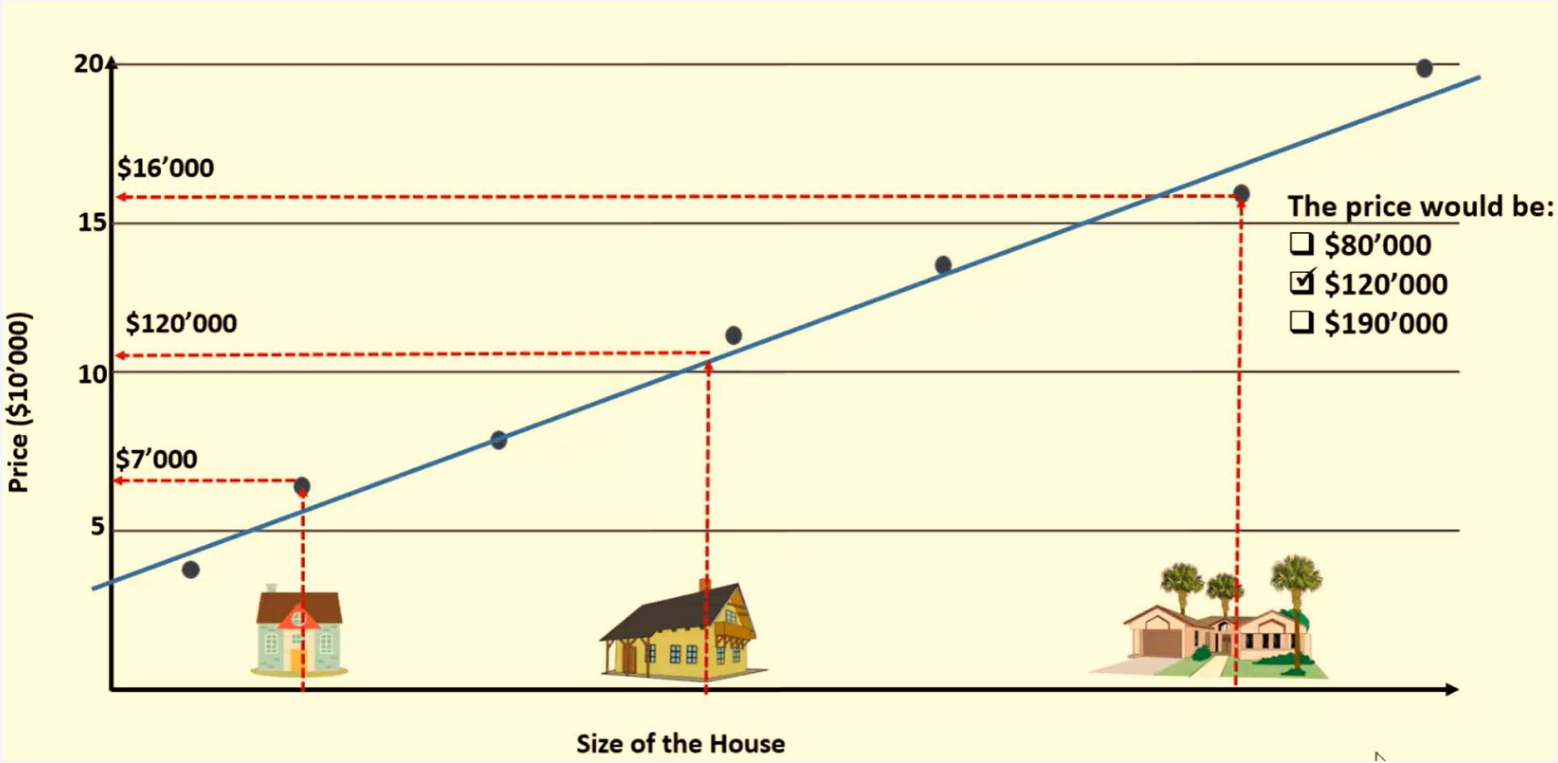
- #Room
- Size
- #Bedroom
- Age
- Address features: Street, Alley, ...
- Size of part1, part2, part3, part4
- Floors
- #Bathrooms
- ...

N number of training data with M featues:

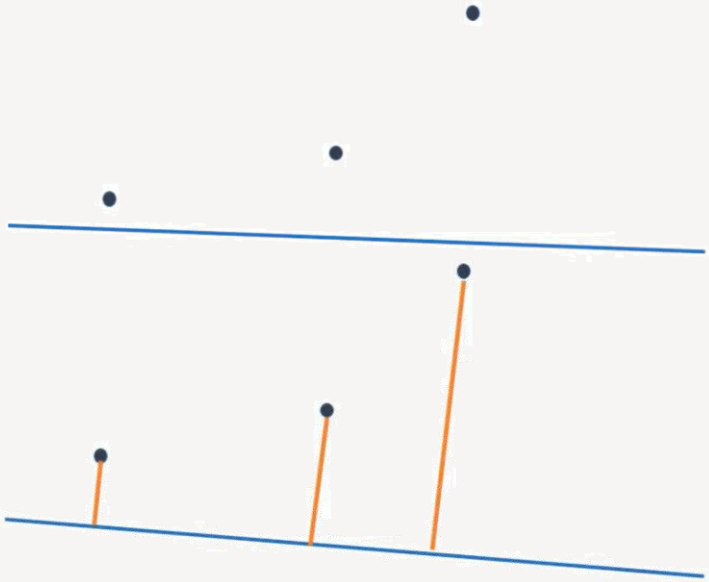
$$A_{N\times M}x_{M\times 1} = b_{N\times 1}$$

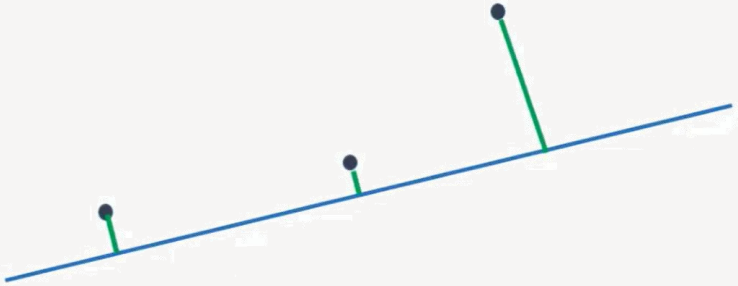
Linear Equation

Least Squares Error Correction



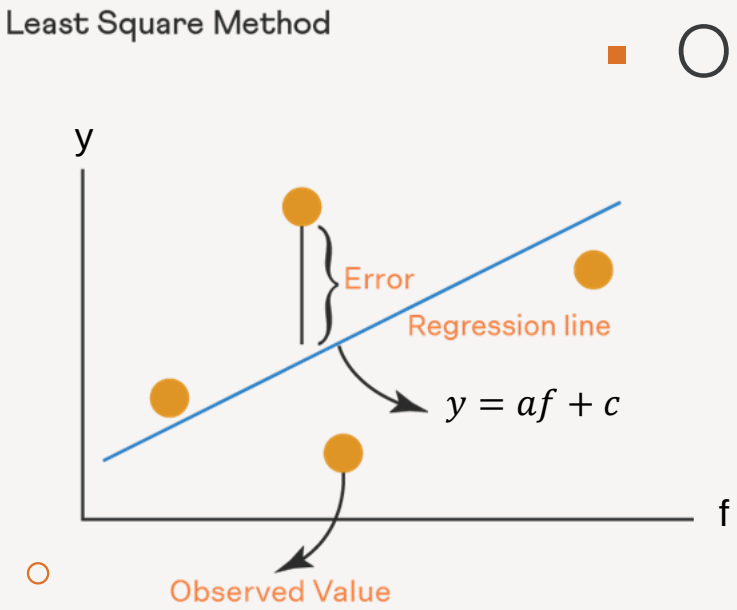
Least Squares Error Correction





- Error 1:
- Error 2:
- Error 3:

Least Squares



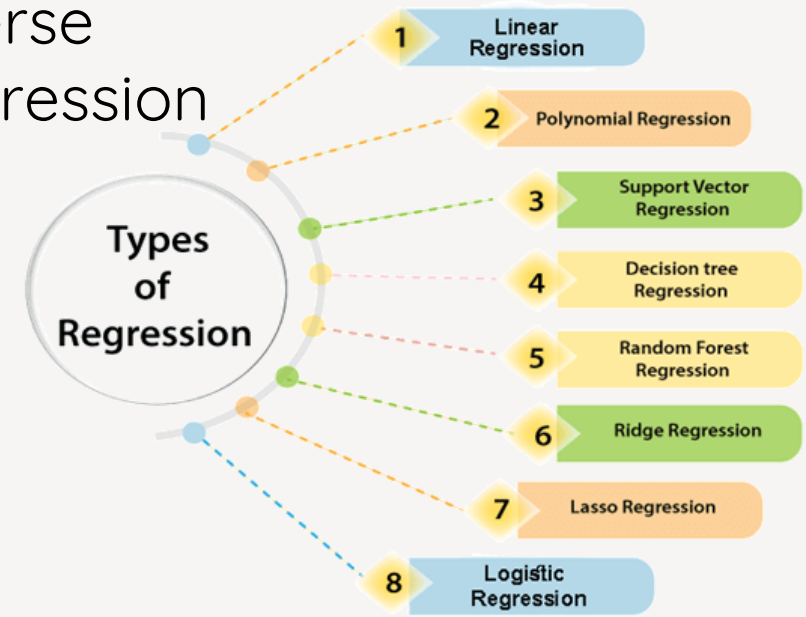
- Objective for “m” features:

$$\hat{y}=a_1 f_1 + a_2 f_2 + \cdots + a_m f_m + c$$

$$\min ||y - \hat{y}||$$

Linear Algebra and Machine Learning Application

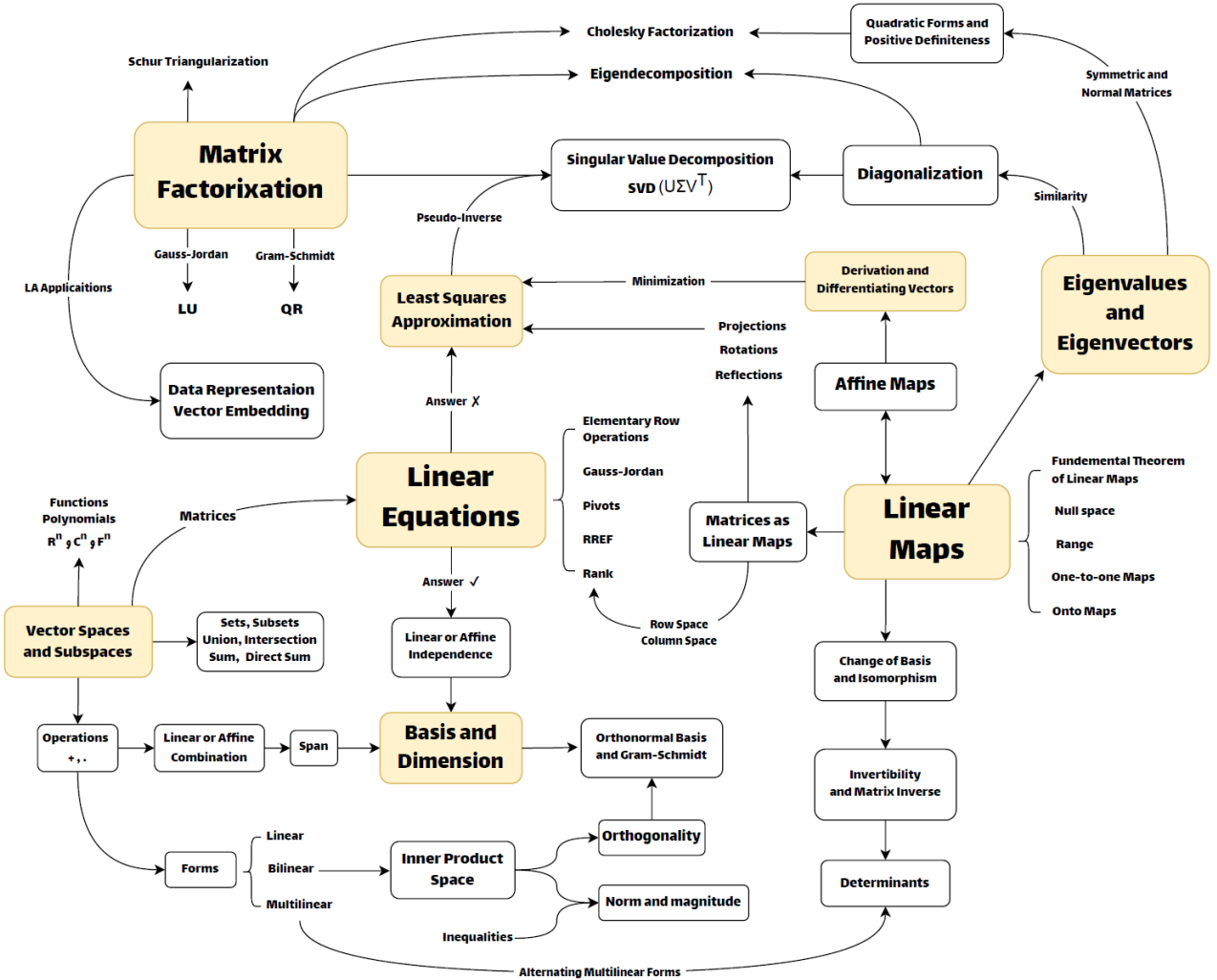
- $Ax = b \quad \rightarrow \quad x = A^{-1}b$ Inverse of Matrix/Pseudo Inverse
- Regression



02

Road Map

Step by Step


Cholesky Factorization
Quadratic Forms and Positive Definiteness
Schur Triangularization
Eigendecomposition
Symmetric and Normal Matrices
Matrix Factorixation
Singular Value Decomposition
SVD $(U\Sigma V^T)$
Diagonalization
Similarity
Pseudo-Inverse
Gauss-Jordan
Gram-Schmidt
LA Applicaitions
LU
QR
Least Squares Approximation
Minimization
Derivation and Differentiating Vectors
Eigenvalues and Eigenvectors
Projections
Rotations
Reflections
Data Representaion Vector Embedding
Affine Maps
Answer ✗
Elementary Row Operations
Gauss-Jordan
Pivots
RREF
Rank
Linear Equations
Fundemental Theorem of Linear Maps
Null space
Range
One-to-one Maps
Onto Maps
Functions Polynomials
R^n , C^n , F^n
Matrices
Matrices as Linear Maps
Linear Maps
Answer ✓
Row Space
Column Space
Vector Spaces and Subspaces
Sets, Subsets Union, Intersection Sum, Direct Sum
Linear or Affine Independence
Change of Basis and Isomorphism
Operations +, .
Linear or Affine Combination
Span
Basis and Dimension
Orthonormal Basis and Gram-Schmidt
Invertibility and Matrix Inverse
Linear
Bilinear
Multilinear
Forms
Inner Product Space
Orthogonality
Norm and magnitude
Determinants
Inequalities
Alternating Multilinear Forms