

# Network Effects and Cascading Behavior

CS224W: Machine Learning with Graphs  
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<http://cs224w.stanford.edu>



# Spreading Through Networks

- **Spreading through networks:**

- Cascading behavior
- Diffusion of innovations
- Network effects
- Epidemics

- **Behaviors that cascade from node to node like an epidemic**

- **Examples:**

- **Biological:**

- Diseases via contagion

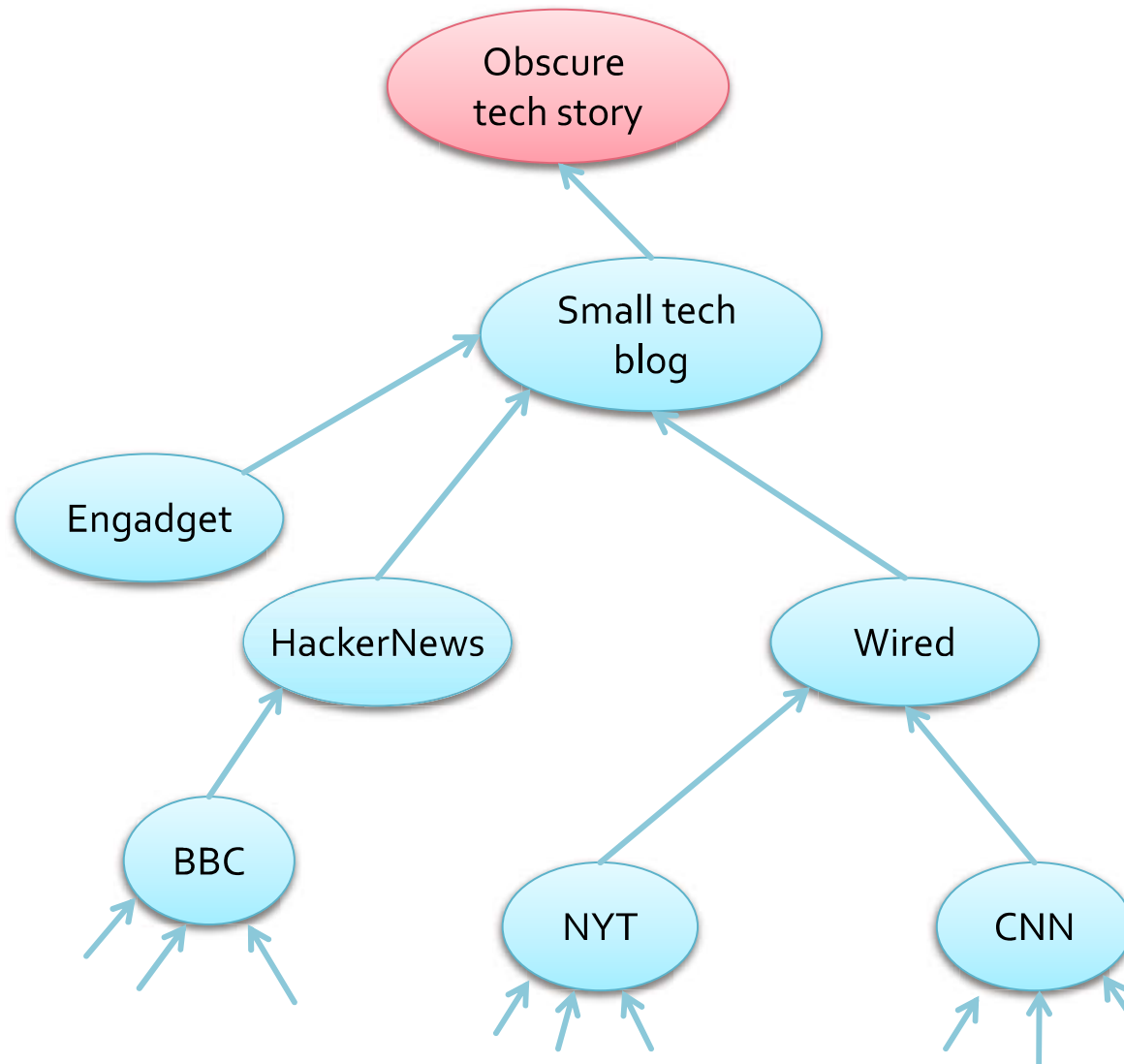
- **Technological:**

- Cascading failures
- Spread of information

- **Social:**

- Rumors, news, new technology
- Viral marketing

# Information Diffusion: Media



# Twitter & Facebook post sharing



**Lada Adamic** shared a link via Erik Johnston.  
January 16, 2013

When life gives you an almost empty jar of nutella, add some ice cream...  
(and other useful tips)



**50 Life Hacks to Simplify your World**  
twistedsifter.com

Life hacks are little ways to make our lives easier. These low-budget tips and trick can help you organize and de-clutter space; prolong and preserve your products; or teach you...

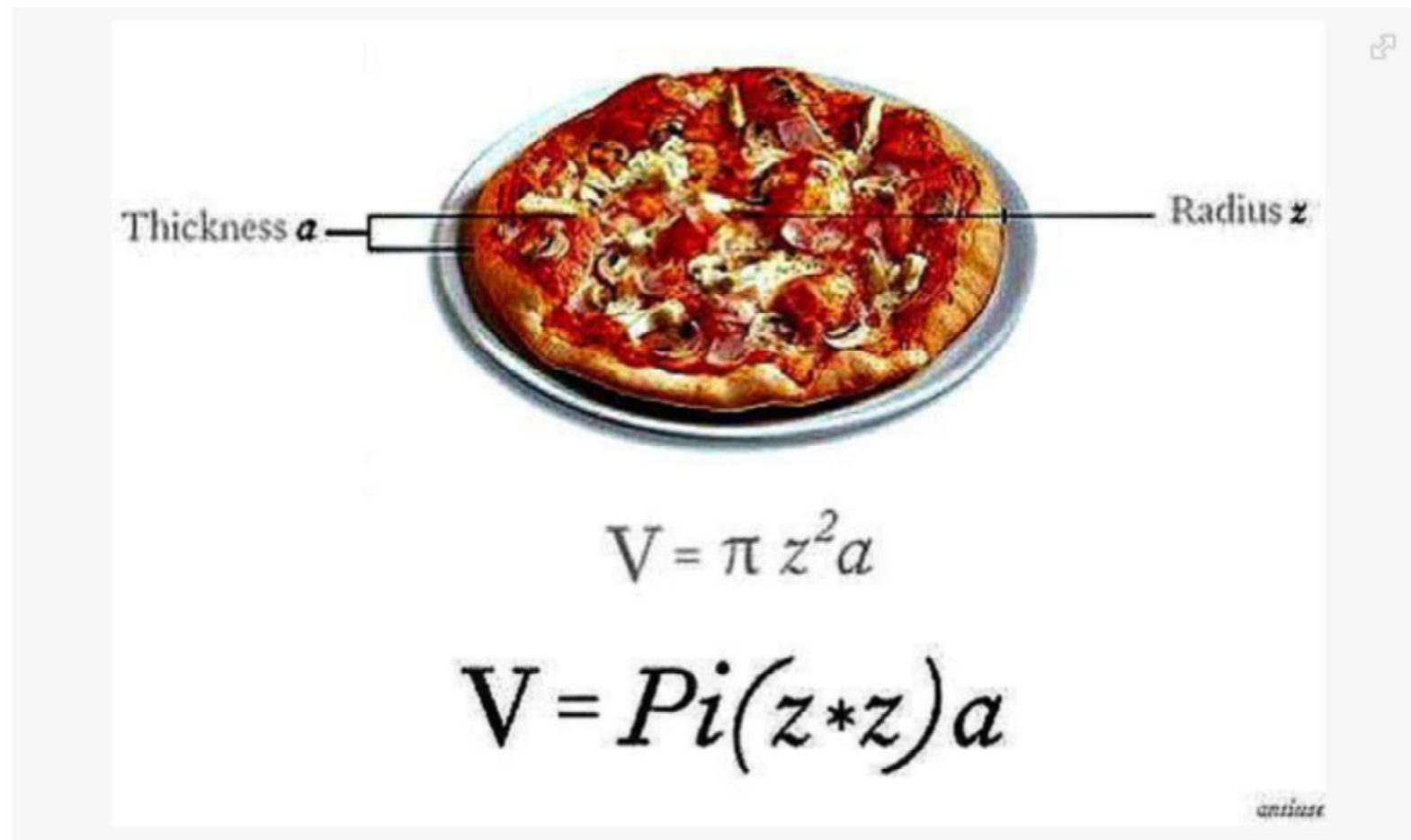
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## Timeline Photos

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**I fucking love science**

Seriously. If you have a pizza with radius "z" and thickness "a", its volume is  $\text{Pi}(z * z)a$ .

Lina von DerStein, Iman Khallaf, 周明佳 and 73,191 others like this.

27,761 shares

Comments

46 of 1,470

Album: [Timeline Photos](#)

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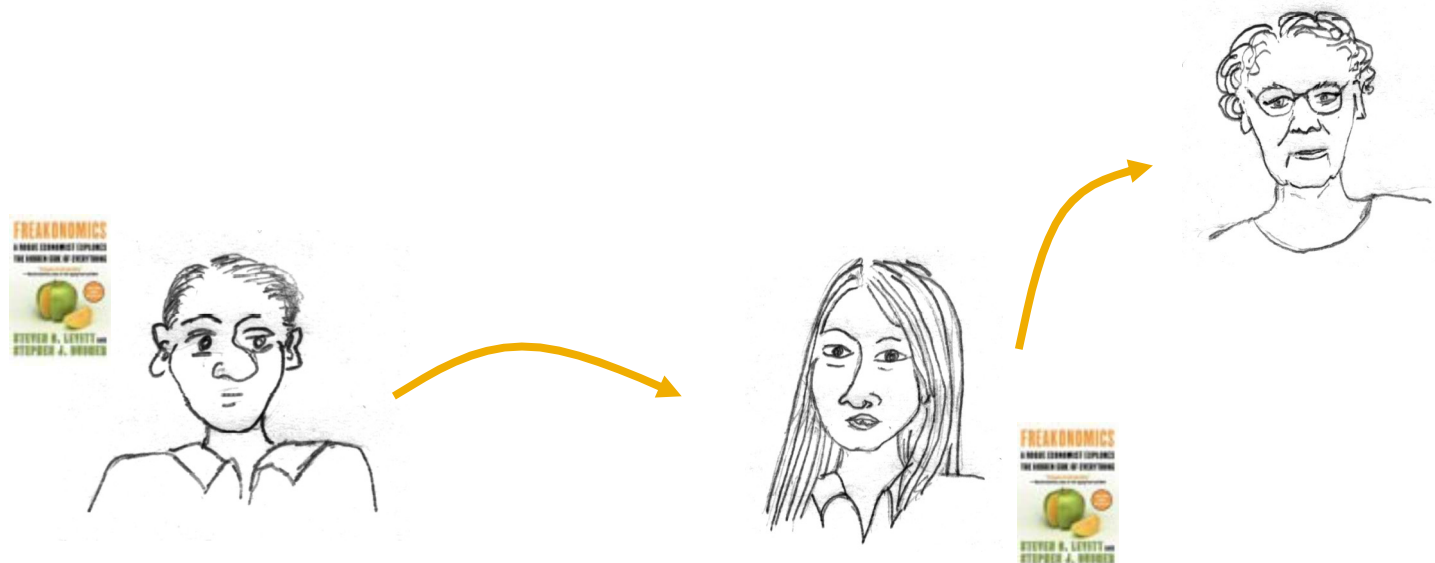
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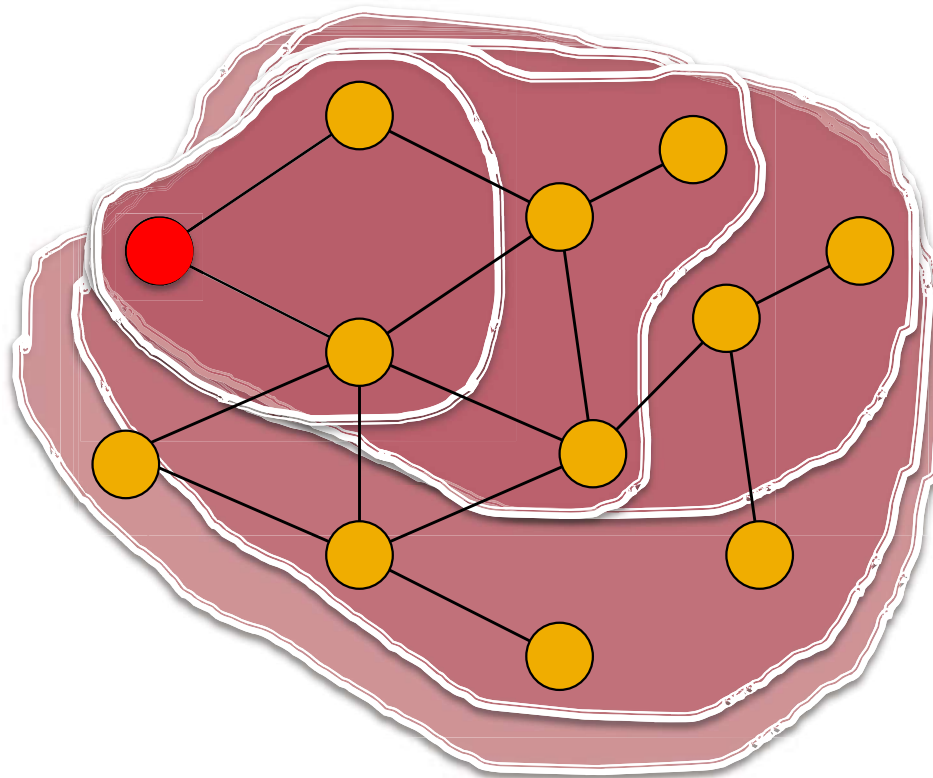
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# Diffusion in Viral Marketing

- **Product adoption:**
  - Senders and followers of recommendations



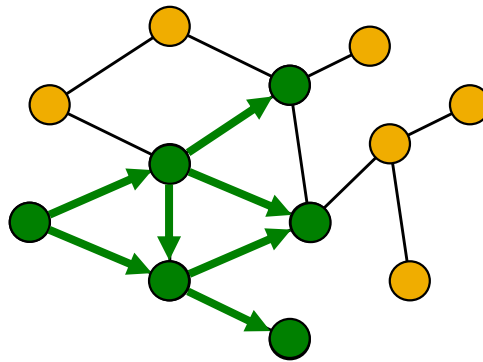
# Spread of Diseases (e.g., Ebola)



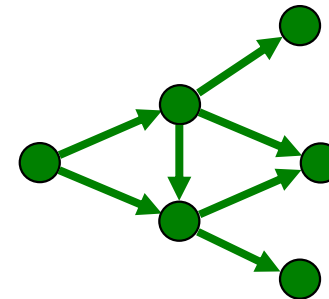


# Network Cascades

- Contagion that spreads over the edges of the network
- It creates a propagation tree, i.e., **cascade**



Network



**Cascade**  
(propagation tree)

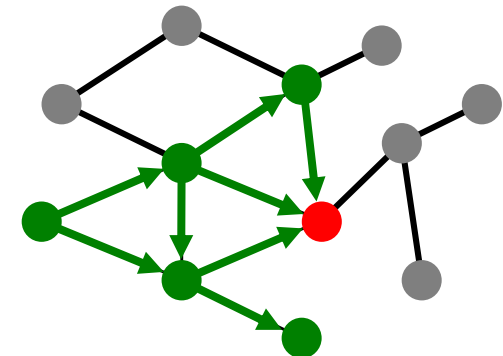
## Terminology:

- What spreads: Contagion
- “Infection” event: Adoption, infection, activation
- Main players: Infected/active nodes, adopters



# How Do We Model Diffusion?

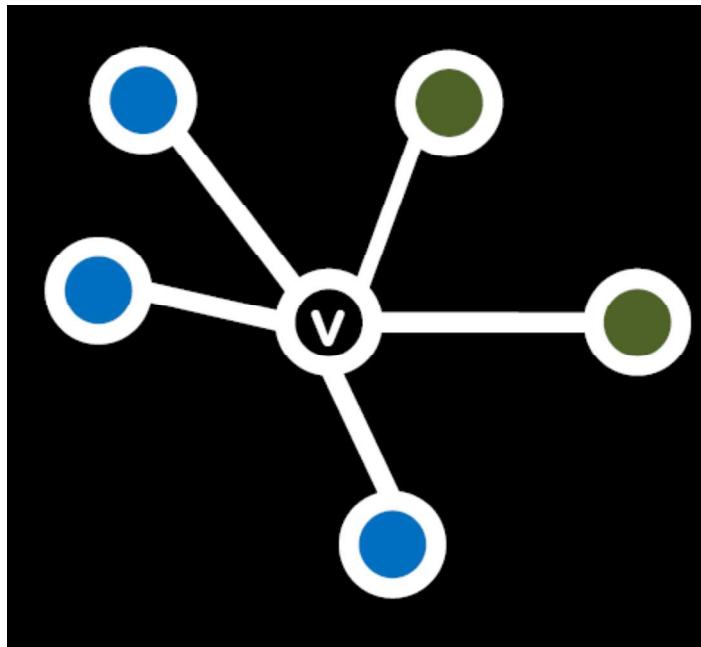
- **Decision based models (today!):**
  - Models of product adoption, decision making
    - A node observes decisions of its neighbors and makes its own decision
  - **Example:**
    - You join demonstrations if  $k$  of your friends do so too
- **Probabilistic models (on Tuesday):**
  - **Models of influence or disease spreading**
    - An infected node tries to “push” the contagion to an uninfected node
  - **Example:**
    - You “catch” a disease with some prob. from each active neighbor in the network



# Decision Based Model of Diffusion

# Game Theoretic Model of Cascades

- **Based on 2 player coordination game**
  - 2 players – each chooses technology A or B
  - Each player can only adopt **one** “behavior”, **A** or **B**
  - **Intuition**: you (node  $v$ ) gain more payoff if your friends have adopted the **same** behavior as you



Local view of the network of node  $v$

# Example: VHS vs. BetaMax



# Example: BlueRay vs. HD DVD



# The Model for Two Nodes

- **Payoff matrix:**

- If both **v** and **w** adopt behavior **A**, they each get payoff  **$a > 0$**
- If **v** and **w** adopt behavior **B**, they each get payoff  **$b > 0$**
- If **v** and **w** adopt the opposite behaviors, they each get **0**

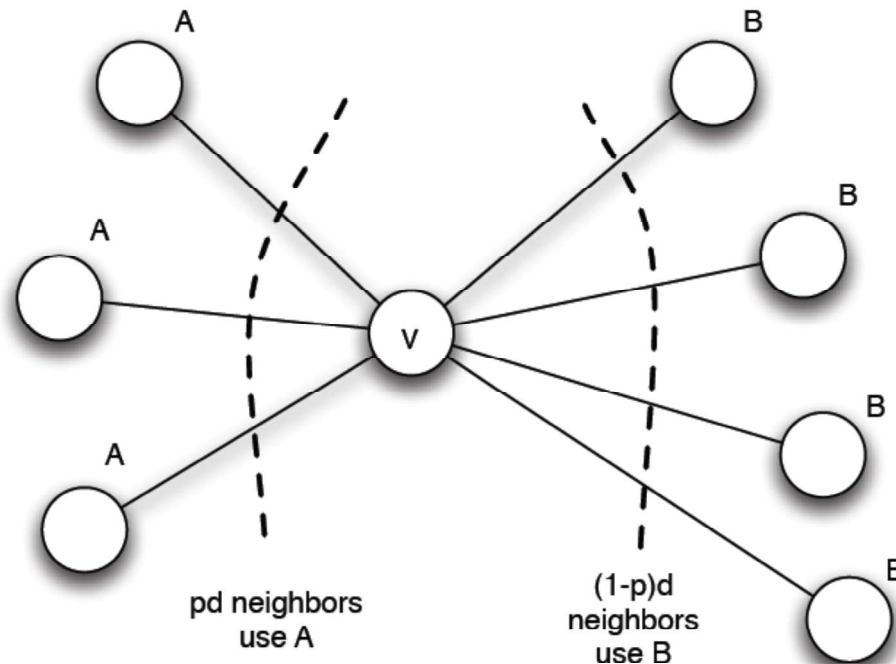


- **In some large network:**

- Each node **v** is playing a copy of the game with each of its neighbors
- **Payoff:** sum of node payoffs over all games



# Calculation of Node $v$



## Threshold:

$v$  chooses **A** if

$$p > \frac{b}{a+b} = q$$

$p$ ... frac.  $v$ 's nbrs. with A

$q$ ... **payoff threshold**

- Let  $v$  have  $d$  neighbors
- Assume fraction  $p$  of  $v$ 's neighbors adopt **A**
  - $Payoff_v = a \cdot p \cdot d$  if  $v$  chooses A
  - $= b \cdot (1-p) \cdot d$  if  $v$  chooses B
- **Thus:  $v$  chooses A if:  $p > q$**



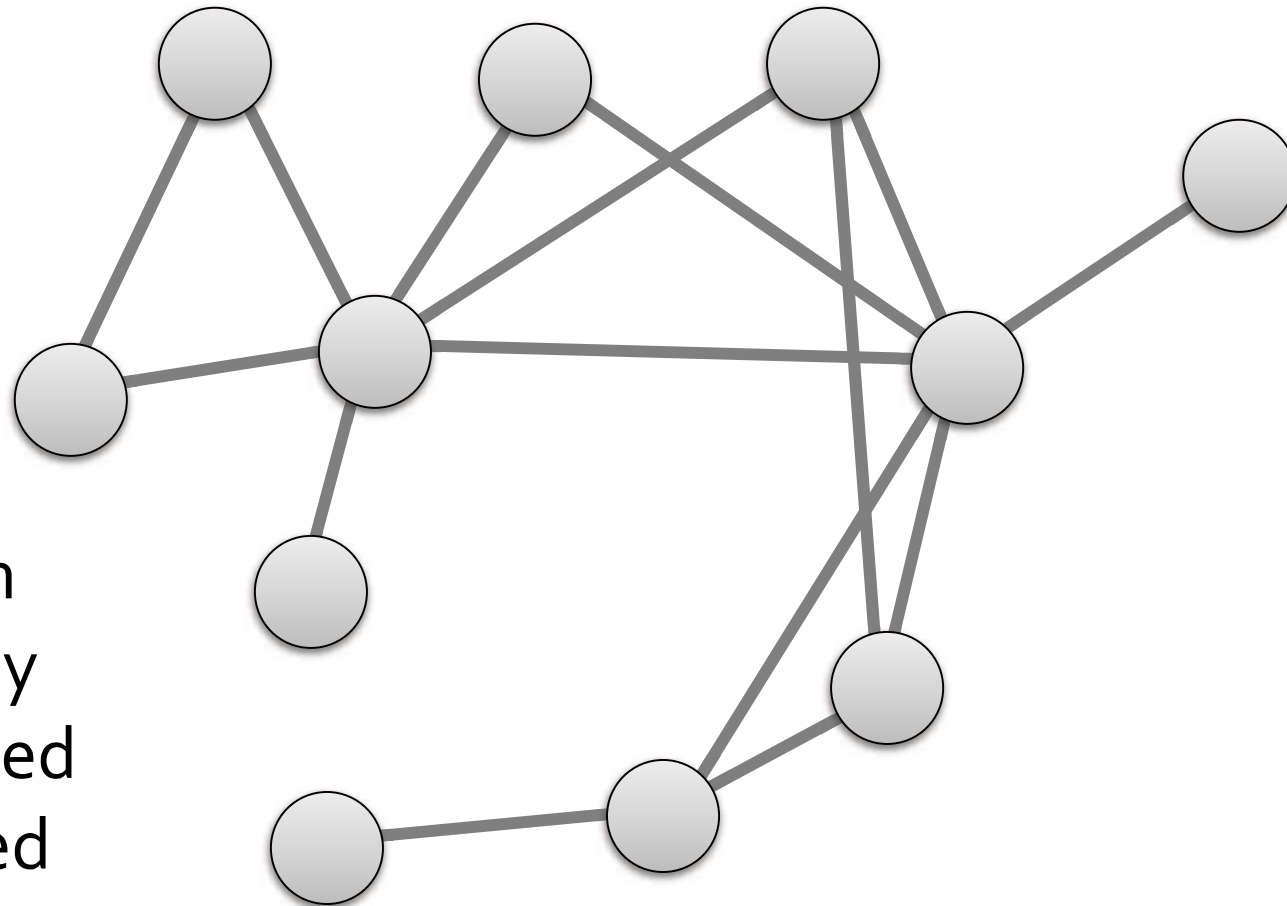
# Example Scenario

## Scenario:

- Graph where everyone starts with all **B**
- Small set **S** of early adopters of **A**
  - Hard-wire **S** – they keep using **A** no matter what payoffs tell them to do
- Assume payoffs are set in such a way that nodes say:  
If more than  $q=50\%$  of my friends take **A**  
I'll also take **A**.  
This means:  $a = b - \epsilon$  ( $\epsilon > 0$ , small positive constant)  
and then  $q=1/2$

# Example Scenario

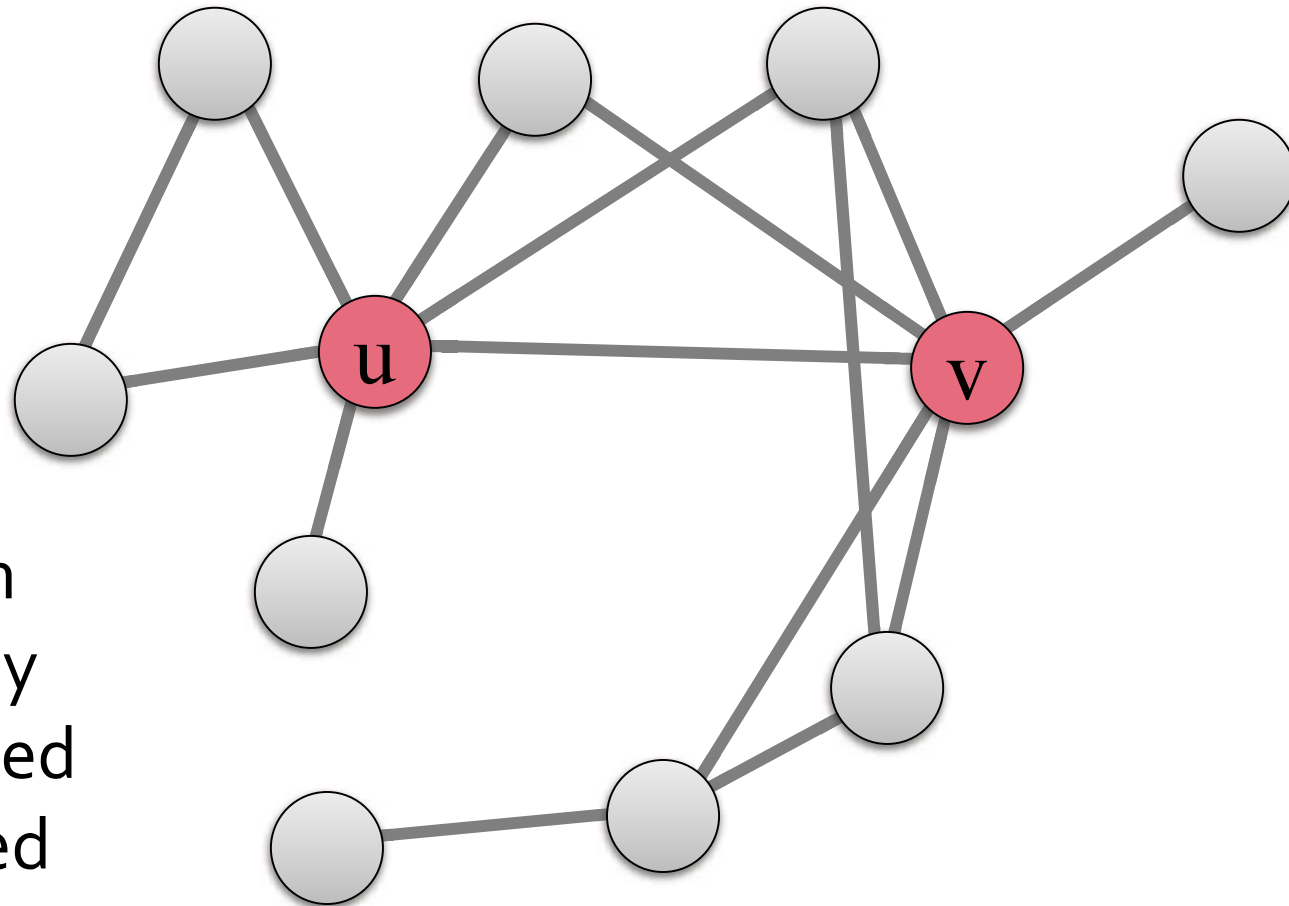
$$S = \{u, v\}$$



If **more** than  
**q=50%** of my  
friends are red  
I'll also be red

# Example Scenario

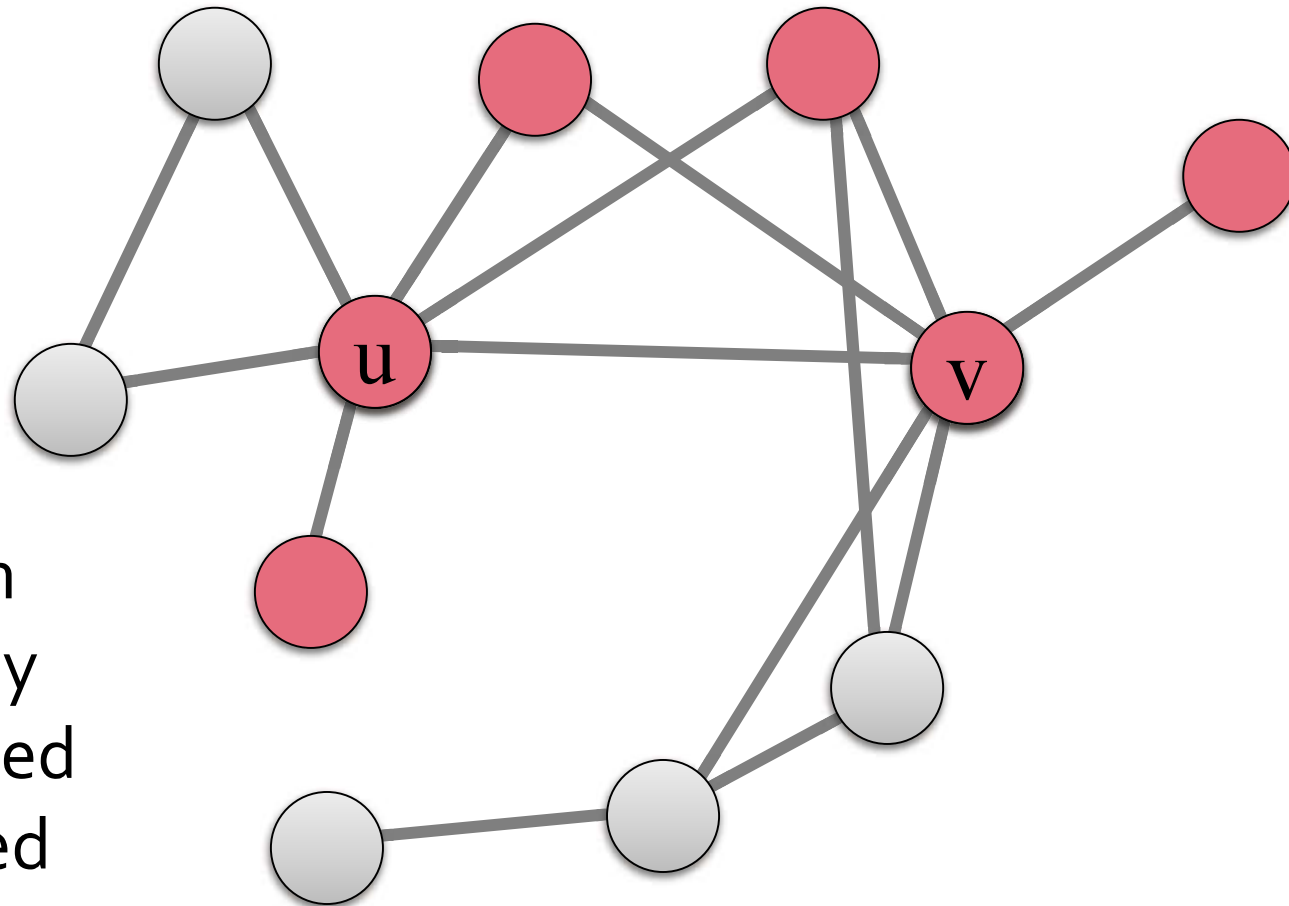
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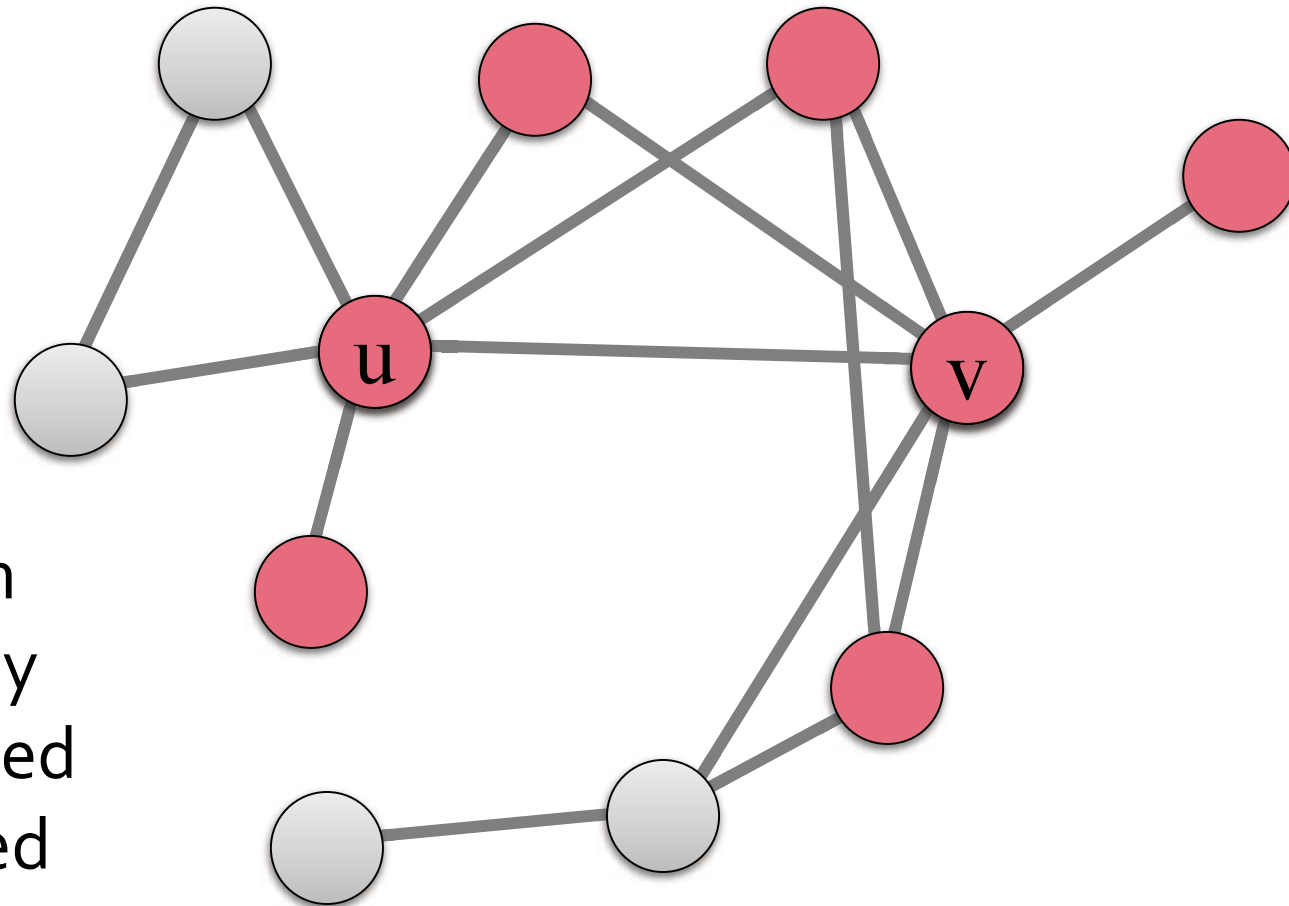
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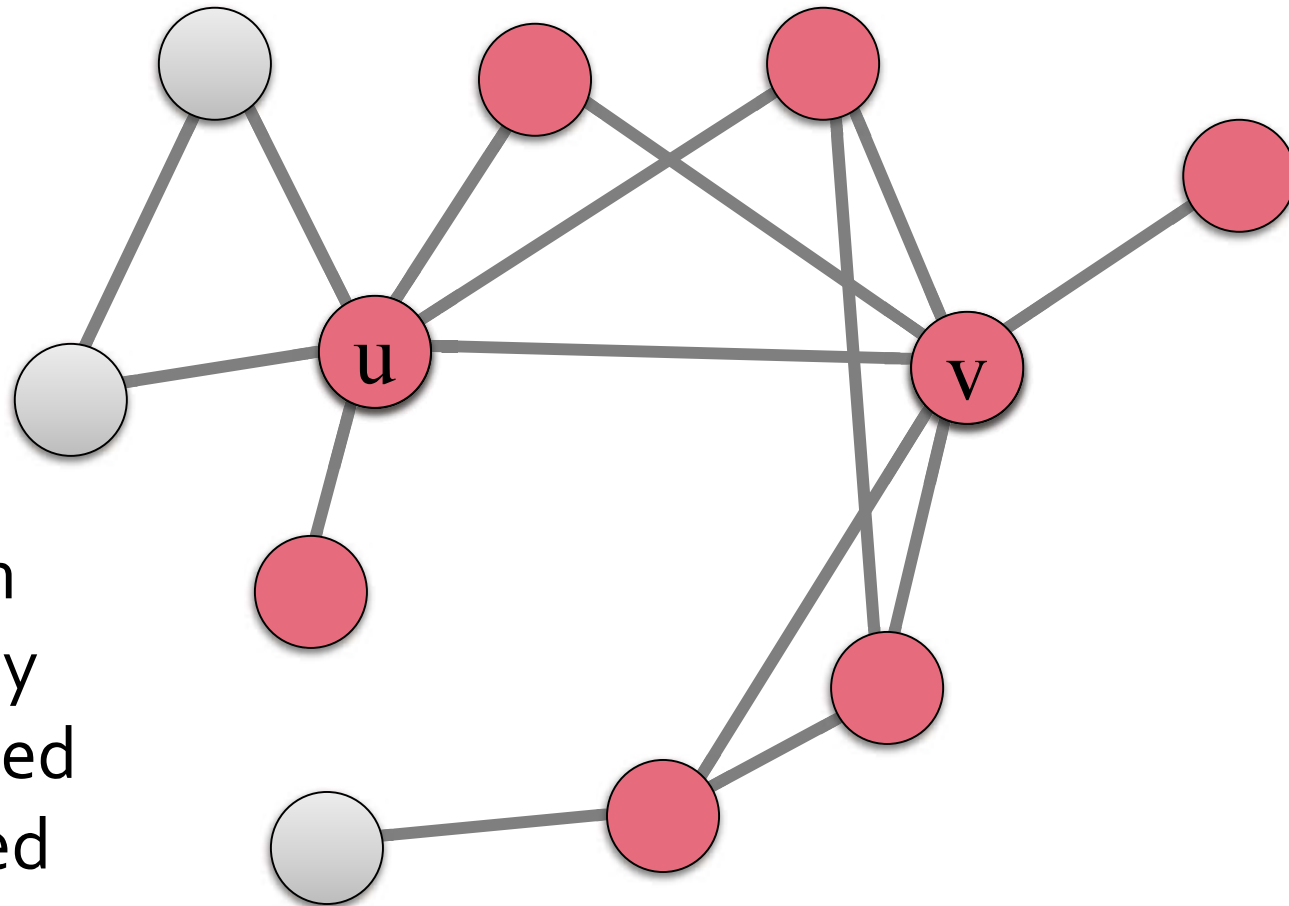
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# Example Scenario

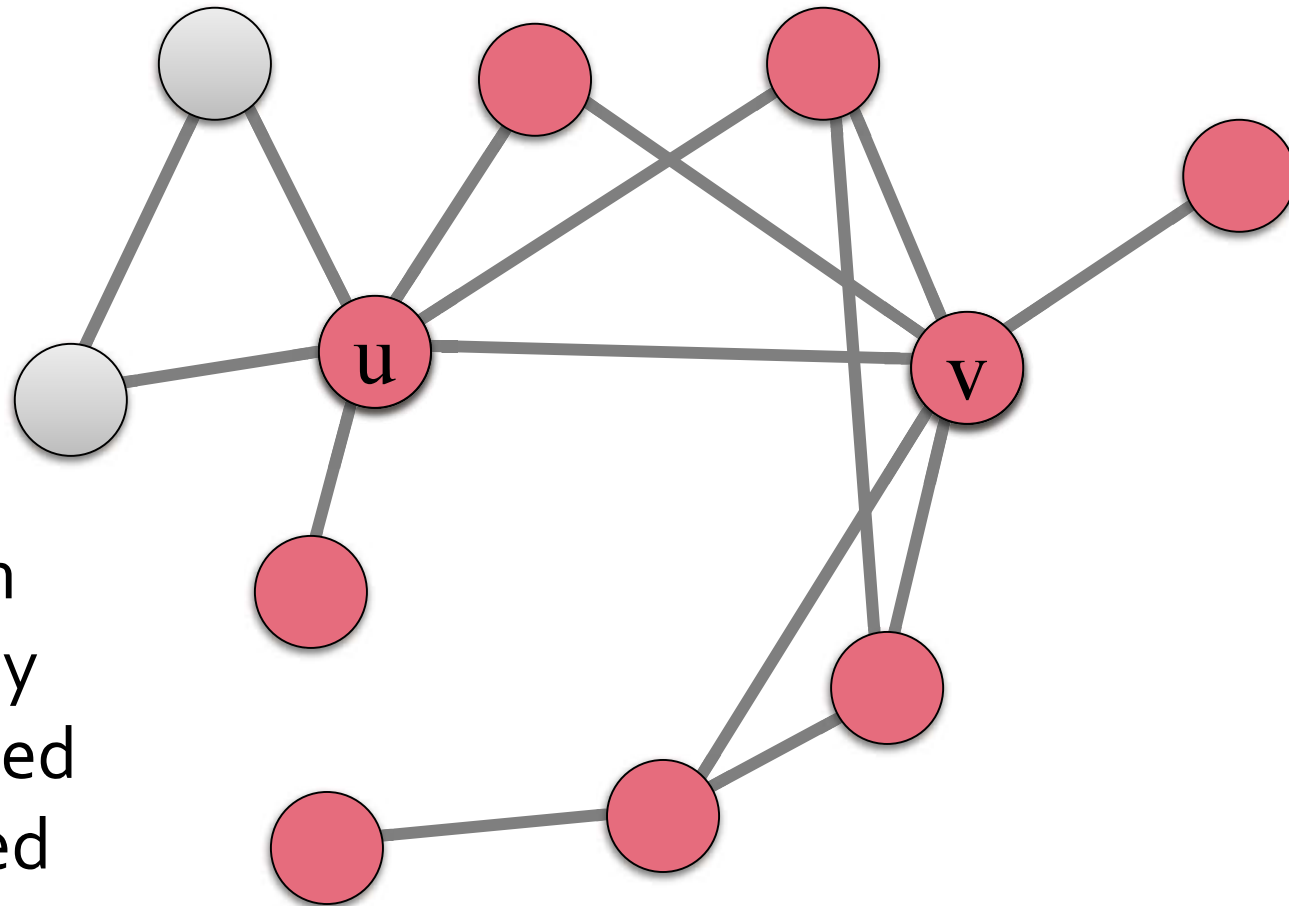
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# Example Scenario

$$S = \{u, v\}$$



If **more** than  
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I'll also be red



# Application: Modeling protest recruitment on social networks

[The Dynamics of Protest Recruitment through an Online Network](#)

Bailon et al. Nature Scientific Reports, 2011

# The Spanish 'Indignados' Movement

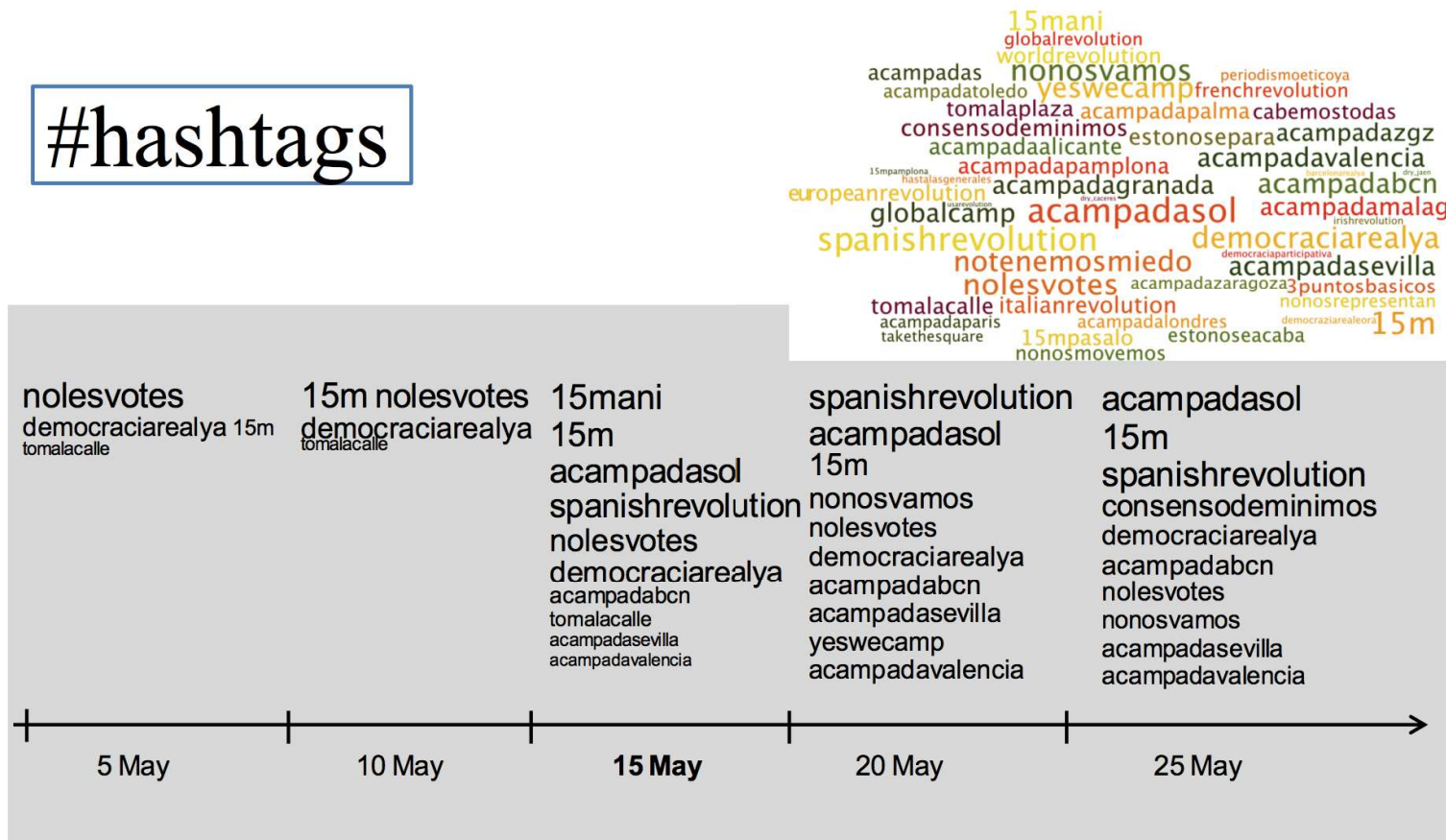
- Anti-austerity protests in Spain May 15-22, 2011 as a response to the financial crisis
- Twitter was used to organize and mobilize users to participate in the protest



# Data collected using hashtags

- Researchers identified 70 hashtags that were used by the protesters

#hashtags



# Dataset

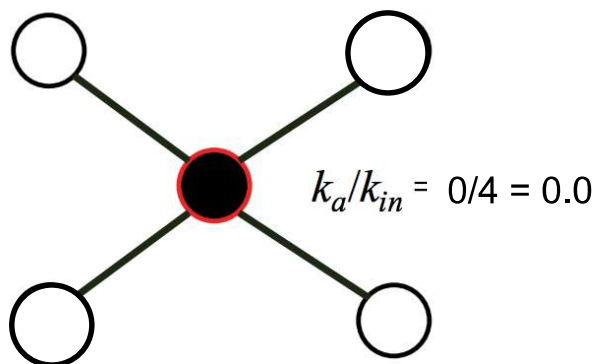
- **70 hashtags were crawled for 1 month period**
  - Number of tweets: 581,750
- **Relevant users:** Any user who tweeted any relevant hashtag and their followers + followees
  - Number of users: 87,569
- **Created two undirected follower networks:**
  1. **Full network:** with all Twitter follow links
  2. **Symmetric network** with only the reciprocal follow links ( $i \rightarrow j$  and  $j \rightarrow i$ )
    - This network represents “strong” connections only.

# Definitions

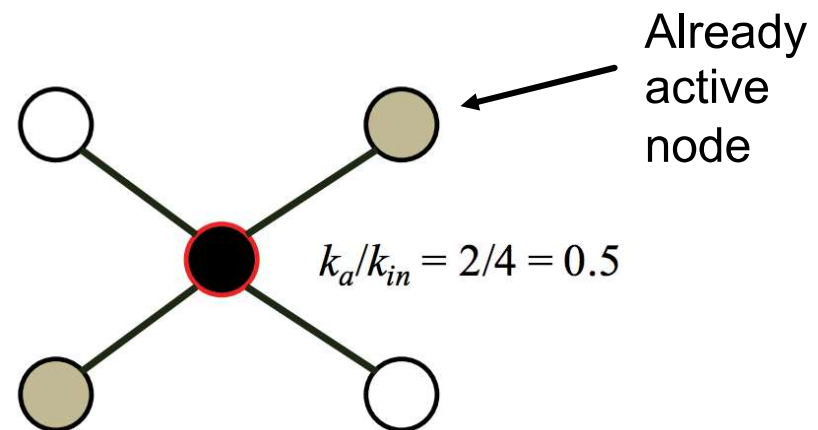
- **User activation time:** Moment when user starts tweeting protest messages
- $k_{in}$  = The total number of neighbors when a user became active
- $k_a$  = Number of active neighbors when a user became active
- **Activation threshold** =  $k_a/k_{in}$ 
  - The fraction of active neighbors at the time when a user becomes active

# Recruitment & Activation Threshold

- If  $k_a/k_{in} \approx 0$ , then the user joins the movement when very few neighbors are active  $\Rightarrow$  no social pressure
- If  $k_a/k_{in} \approx 1$ , then the user joins the movement after most of its neighbors are active  $\Rightarrow$  high social pressure



No social pressure for middle node to join

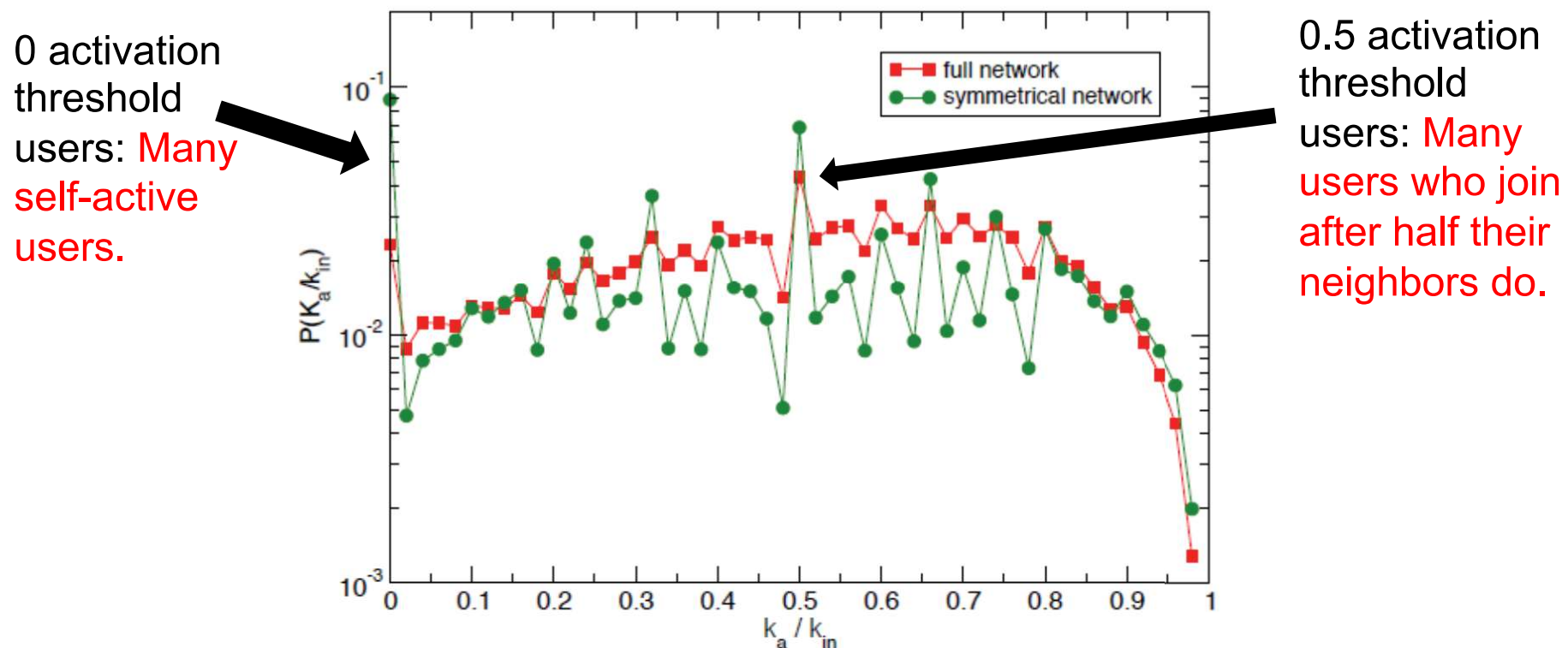


Non-zero social pressure for middle node to join



# Distribution of activation thresholds

- Mostly uniform distribution of activation threshold in both networks, except for two local peaks

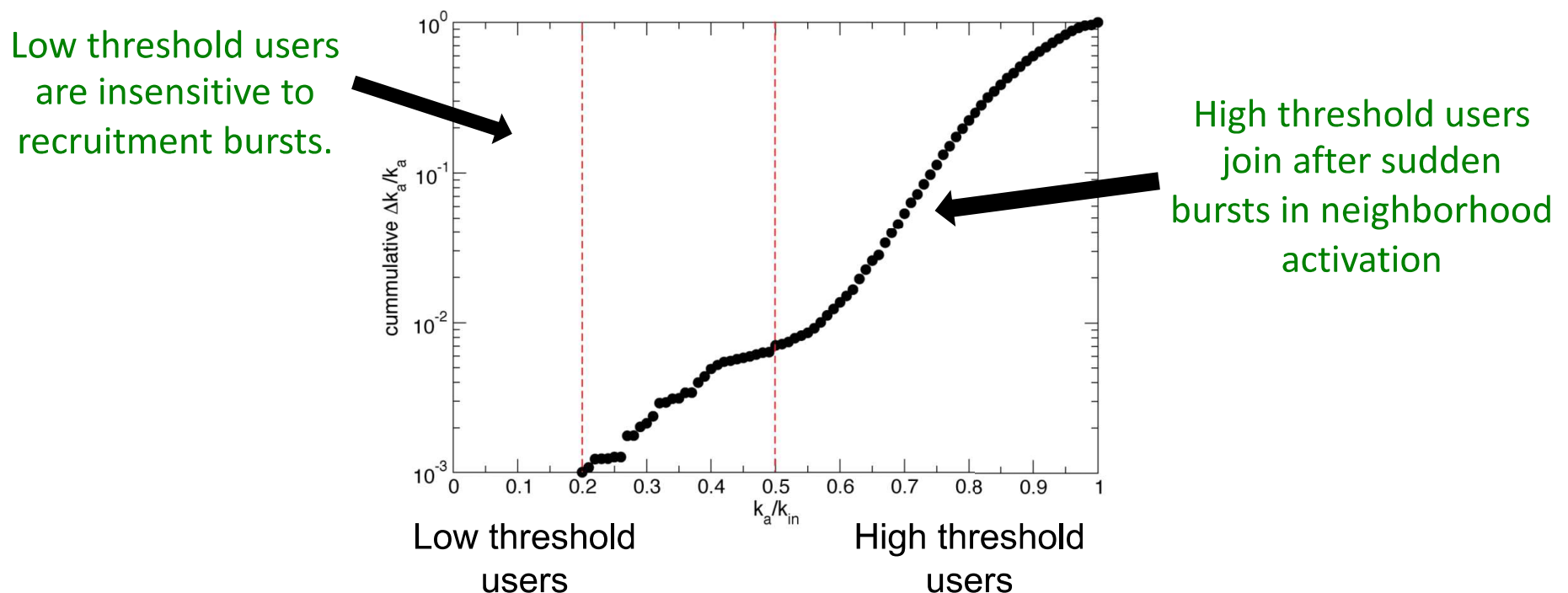




# Effect of neighbor activation time

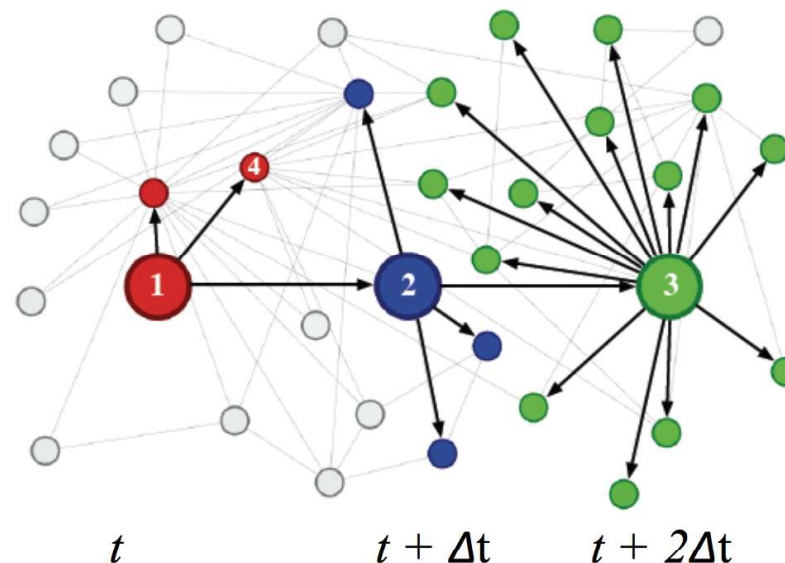
- **Hypothesis:** If several neighbors become active in a short time period, then a user is more likely to become active
- **Method:** Calculate the burstiness of active neighbors as

$$\Delta k_a / k_a = (k_a^{t+1} - k_a^t) / k_a^{t+1}$$



# Information cascades

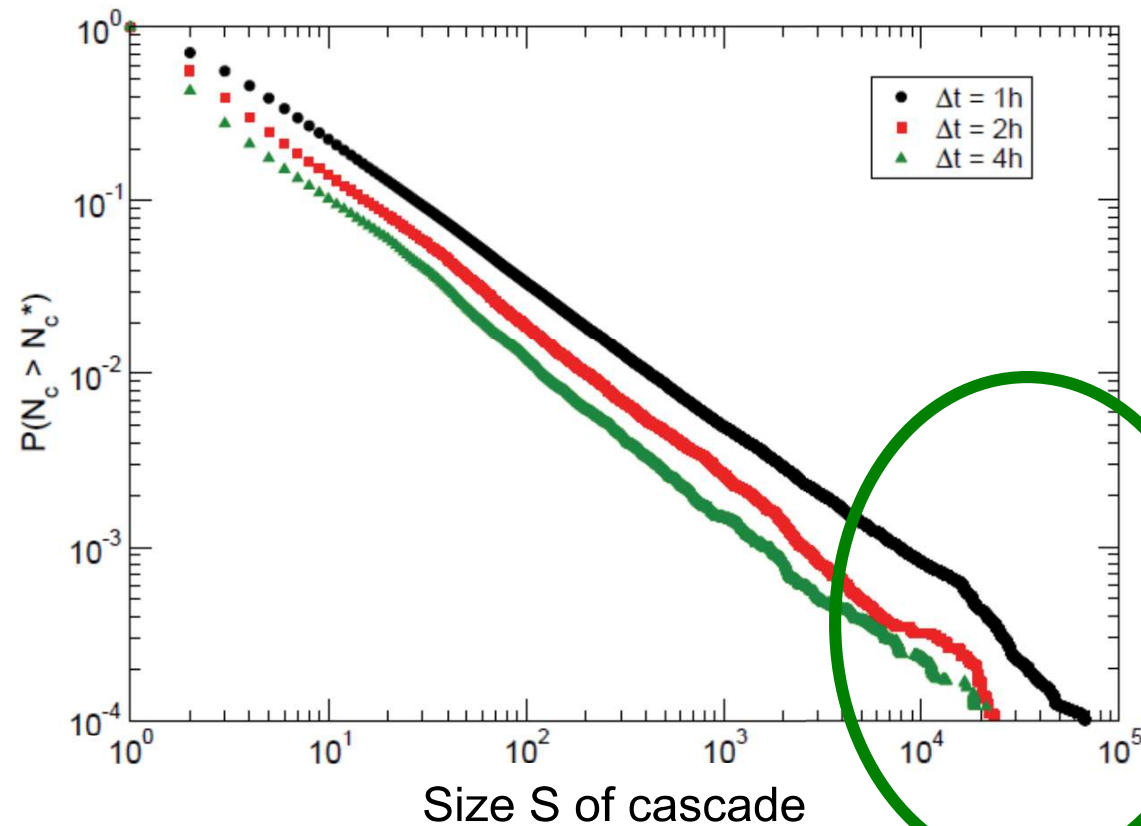
- No cascades are given in the data
- **So cascades were identified as follows:**
  - If a user tweets a message at time  $t$  and one of its followers tweets a message in  $(t, t + \Delta t)$ , then they form a cascade.
  - E.g.,  $1 \rightarrow 2 \rightarrow 3$  below form a cascade:



# Size of information cascades

- **Size** = number of nodes in the cascade
- **Most cascades are small:**

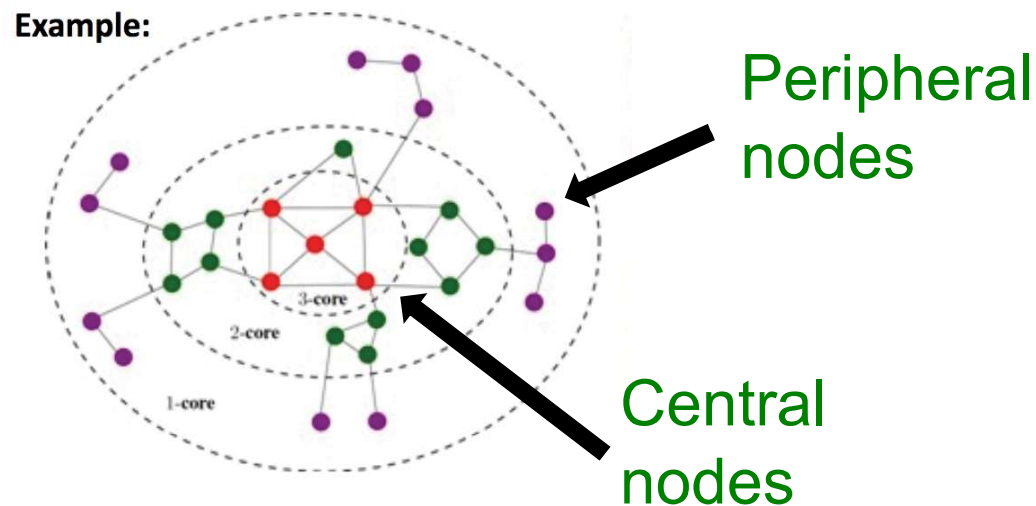
Fraction of cascades with size at least  $S$



Successful cascades

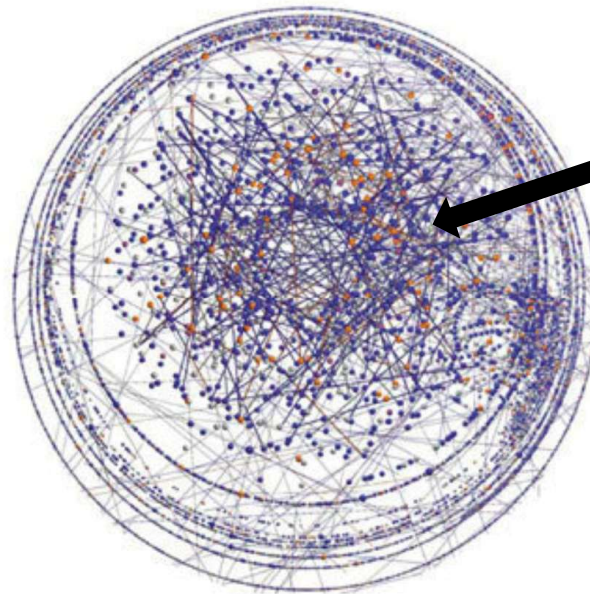
# Who starts successful cascades?

- Are starters of successful cascades more central in the network?
- **Method:**  $k$ -core decomposition
  - $k$ -core: biggest connected subgraph where every node has at least degree  $k$
  - Method: repeatedly remove all nodes with degree less than  $k$
  - Higher  $k$ -core number of a node means it is more central



# Who starts the successful cascades?

- K-core decomposition of follow network
  - Red nodes start successful cascades
  - Red nodes have higher  $k$ -core values
    - So, successful cascade starters are central and connected to equally well connected users



Successful  
cascade starters  
are central (higher  
 $k$ -core number)

# Summary: Cascades on Twitter

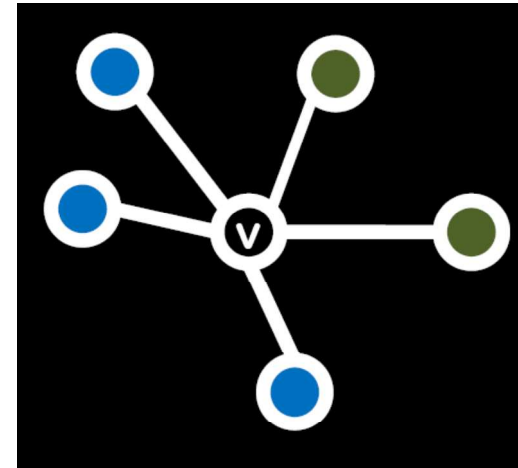
- Uniform activation threshold for users, with two local peaks
- Most cascades are short
- Successful cascades are started by central (more core) users

# Models of Cascading Behavior

- So far:

- Decision Based Models**

- Utility based
    - Deterministic
    - “Node” centric: A node observes decisions of its neighbors and makes its own decision



- **Next: Extending decision based models to multiple contagions**



**Extending the Model:  
Allow People to Adopt A and B**

# Extending the model

- **So far:**

- Behaviors **A** and **B** compete
- Can only get utility from neighbors of same behavior: **A-A** get **a**, **B-B** get **b**, **A-B** get **0**

- **For example:**

- **Using Skype vs. WhatsApp**

- Can only talk using the same software

- **Having a VHS vs. BetaMax player**

- Can only share tapes with people using the same type of tape

- **But one can buy 2 players or install 2 programs**

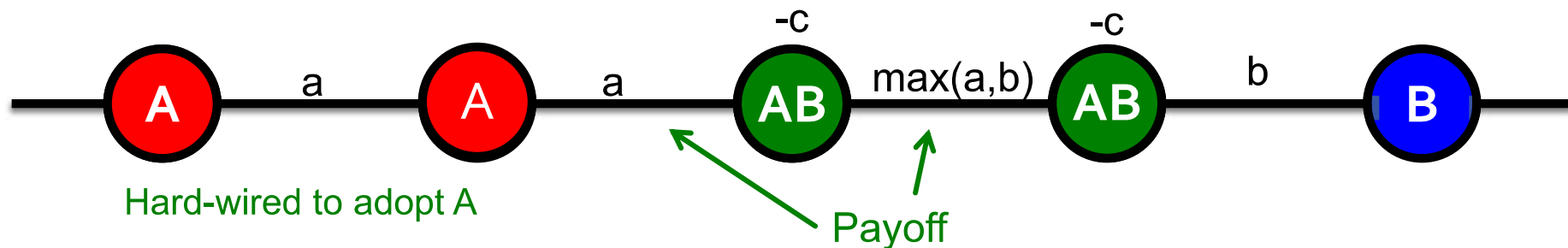


# Cascades & Compatibility

- **So far:**
  - Behaviors **A** and **B** compete
  - Can only get utility from neighbors of same behavior: **A-A** get **a**, **B-B** get **b**, **A-B** get **0**
- **Let's add an extra strategy "AB"**
  - **AB-A** : gets **a**
  - **AB-B** : gets **b**
  - **AB-AB** : gets **max(a, b)**
  - **Also:** Some **cost c** for the effort of maintaining both strategies (summed over all interactions)
    - Note: a given node can receive **a** from one neighbor and **b** from another by playing AB, which is why it could be worth the cost **c**

# Cascades & Compatibility: Model

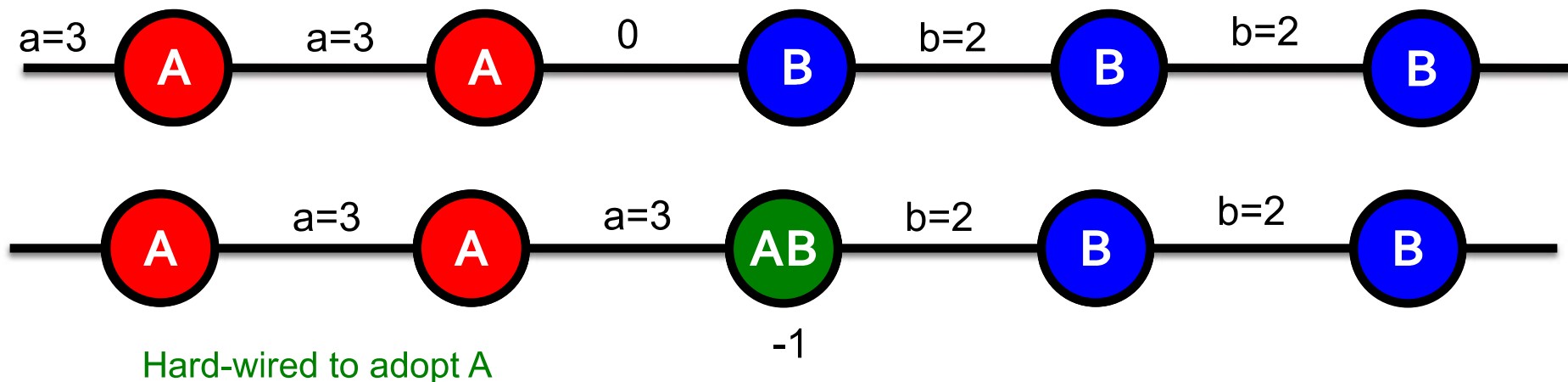
- Every node in an infinite network starts with **B**
- Then a finite set **S** initially adopts **A**
- Run the model for  $t=1,2,3,\dots$ 
  - Each node selects behavior that will optimize payoff (given what its neighbors did in at time  $t-1$ )



- How will nodes switch from **B** to **A** or **AB**?

# Example: Path Graph (1)

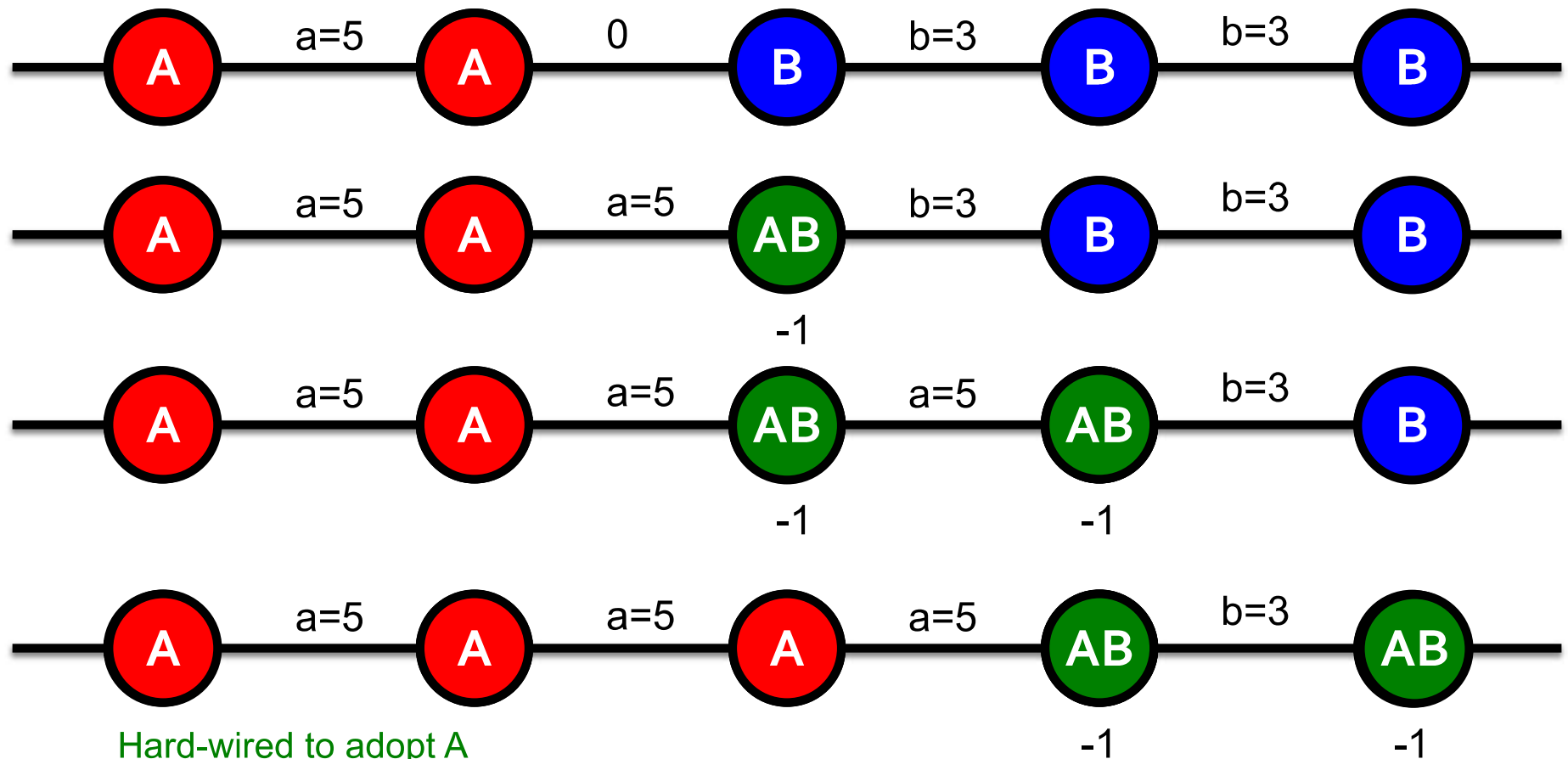
- **Path graph:** Start with Bs,  $a > b$  (A is better)
- **One node switches to A – what happens?**
  - With just A, B: A spreads if  $a > b$
  - With A, B, AB: Does A spread?
- **Example:  $a=3, b=2, c=1$**



**Cascade stops**

# Example: Path Graph (2)

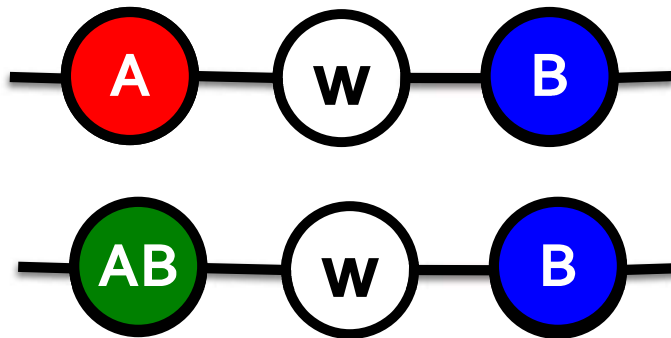
- Example:  $a=5$ ,  $b=3$ ,  $c=1$



**Cascade never stops!**

# What about in a general case?

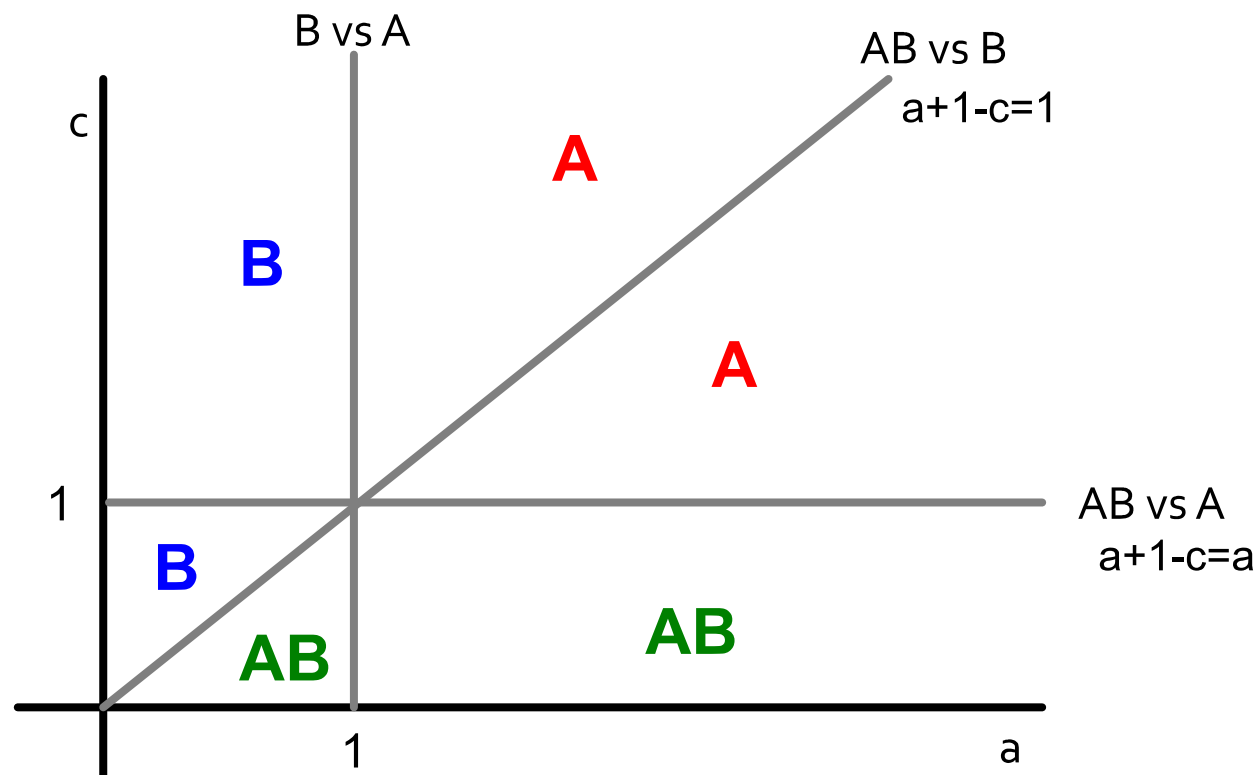
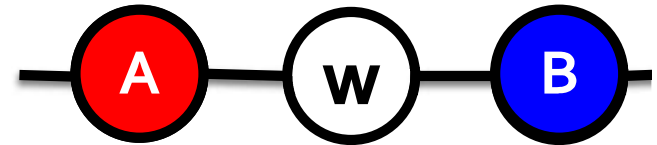
- Let's solve the model in a general case:
  - Infinite path, start with all Bs
  - Payoffs for  $w$ :  $A:a$ ,  $B:1$ ,  $AB:a+1-c$
- For what pairs  $(c,a)$  does A spread?
  - We need to analyze two cases for node  $w$ : Based on the values of  $a$  and  $c$ , what would  $w$  do?





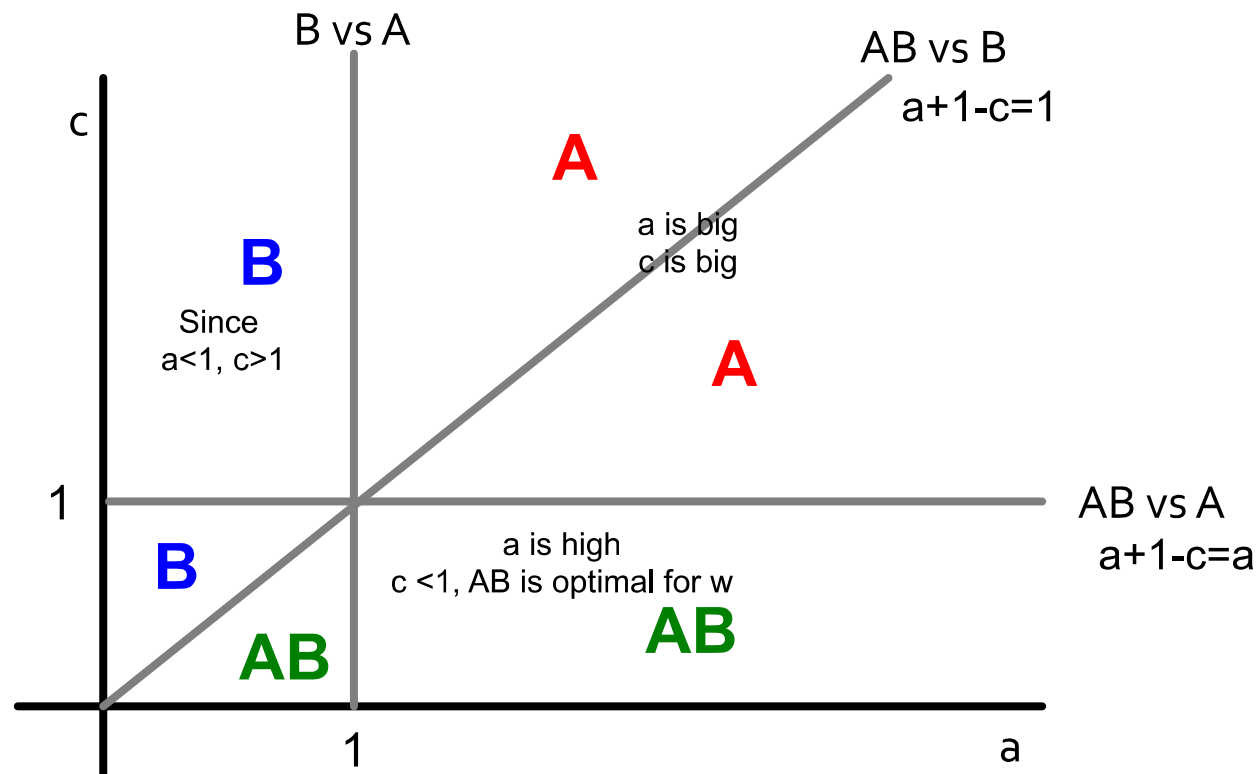
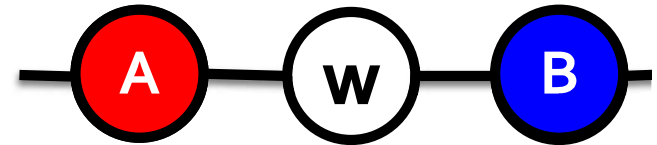
# For what pairs $(c, a)$ does A spread?

- Infinite path, start with Bs
- Payoffs for  $w$ : A: $a$ , B: $1$ , AB: $a+1-c$
- What does node  $w$  adopt?



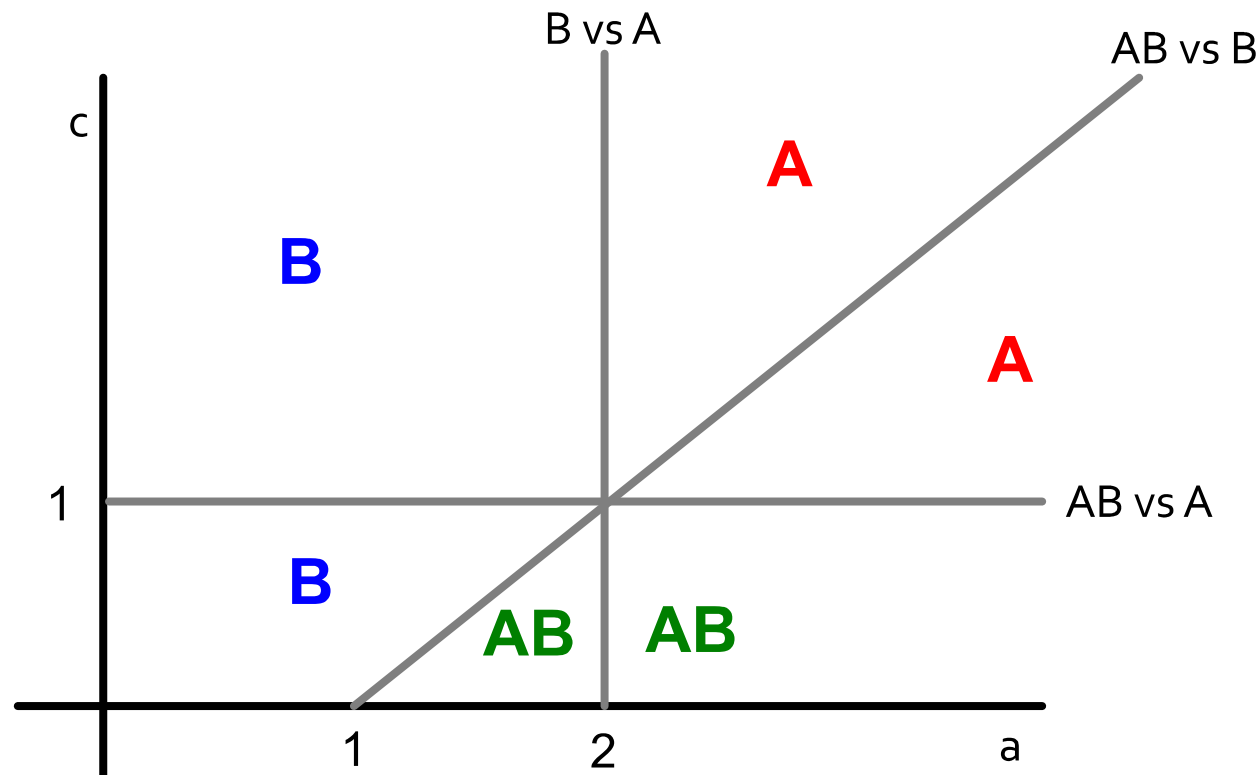
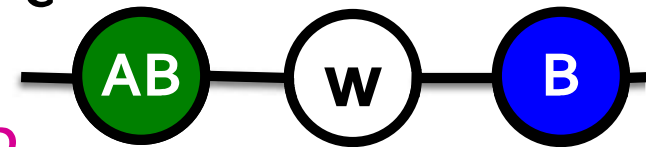
# For what pairs $(c,a)$ does A spread?

- Infinite path, start with Bs
- Payoffs for  $w$ : A: $a$ , B: $1$ , AB: $a+1-c$
- What does node  $w$  in A- $w$ -B do?



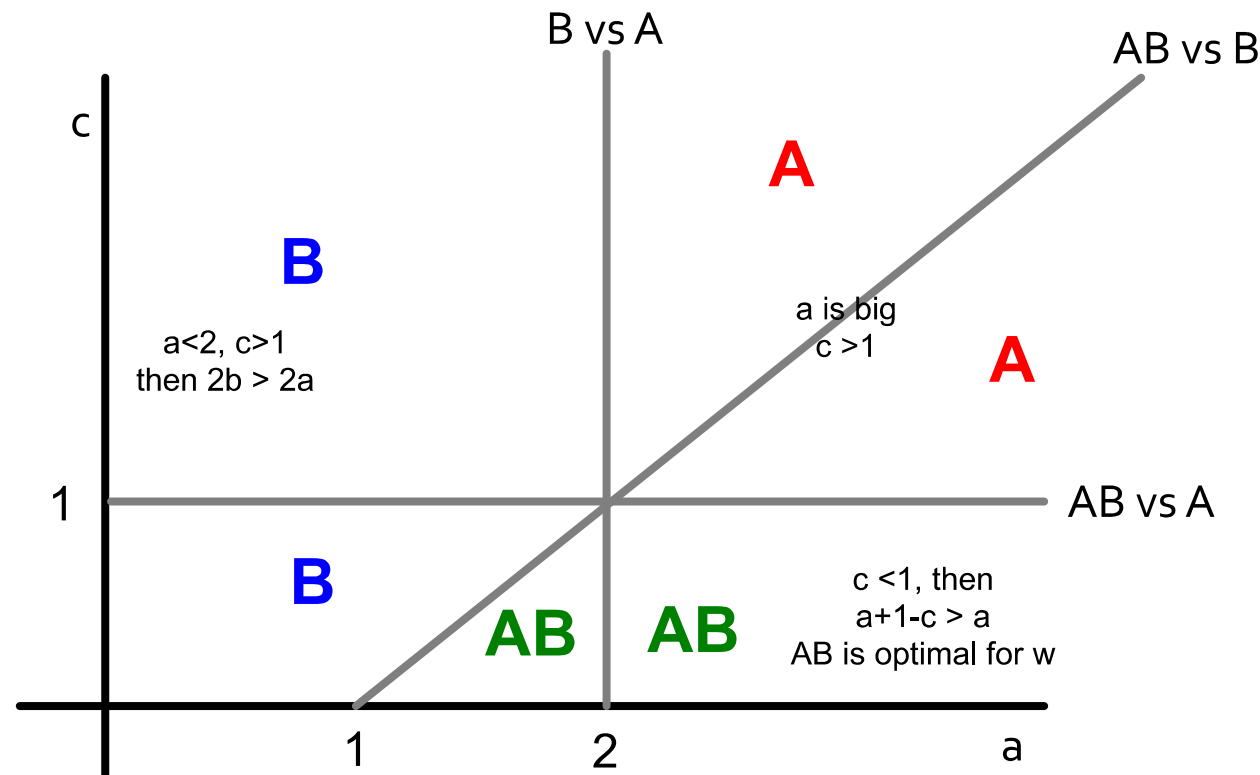
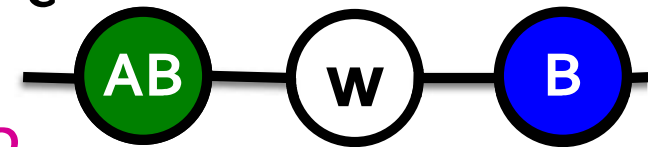
# For what pairs $(c,a)$ does A spread?

- Same reward structure as before but now payoffs for  $w$  change:  $A:a$ ,  $B:1+1$ ,  $AB:a+1-c$
- Notice: Now also  $AB$  spreads
- What does node  $w$  in  $AB-w-B$  do?



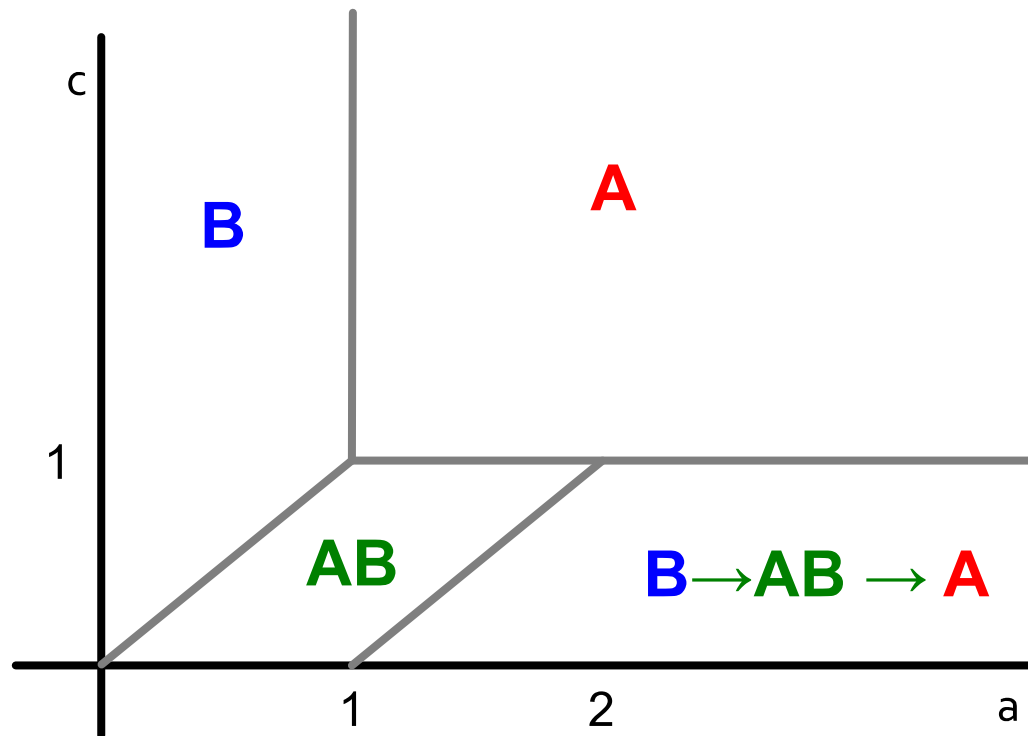
# For what pairs $(c,a)$ does A spread?

- Same reward structure as before but now payoffs for  $w$  change: A: $a$ , B: $1+1$ , AB: $a+1-c$
- Notice: Now also AB spreads
- What does node  $w$  in AB- $w$ -B do?



# For what pairs $(c,a)$ does A spread?

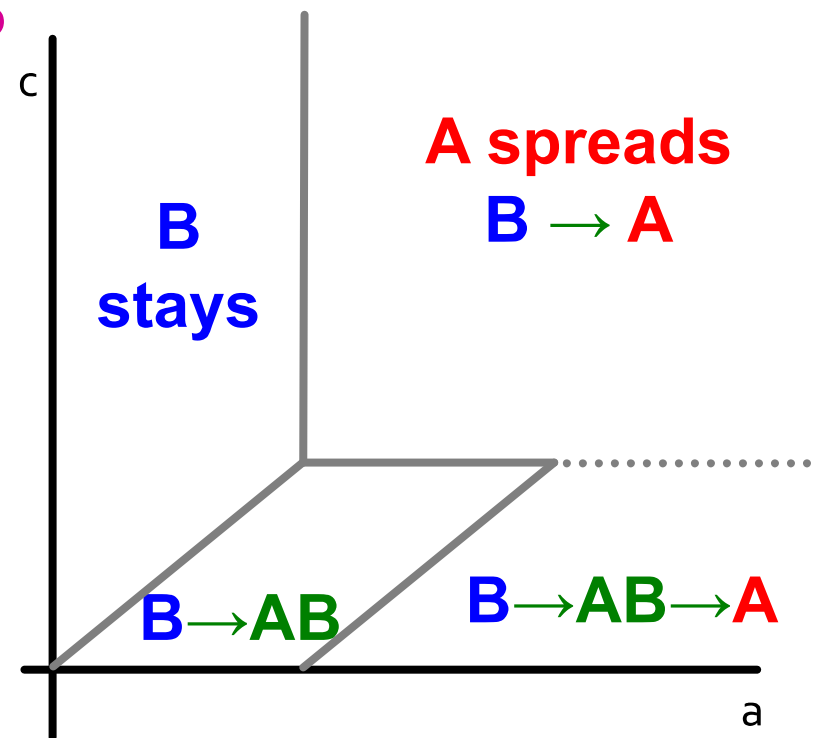
- Joining the two pictures:



# Lesson

- **B is the default throughout the network until new/better A comes along. What happens?**

- **Infiltration:** If **B** is **too compatible** then people will take on both and then drop the worse one (**B**)
- **Direct conquest:** If **A** makes itself **not compatible** – people on the border must choose. They pick the better one (**A**)
- **Buffer zone:** If you choose an optimal level then you keep a static “buffer” between **A** and **B**

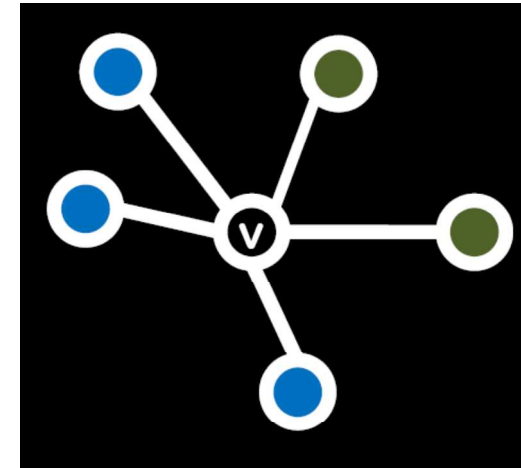


# Models of Cascading Behavior

## ■ So far:

### Decision Based Models

- Utility based
- Deterministic
- “Node” centric: A node observes decisions of its neighbors and makes its own decision
- Require us to know too much about the data



## ■ Next: Probabilistic Models

- Lets you do things by observing data
- **Limitation:** we can't model causality

